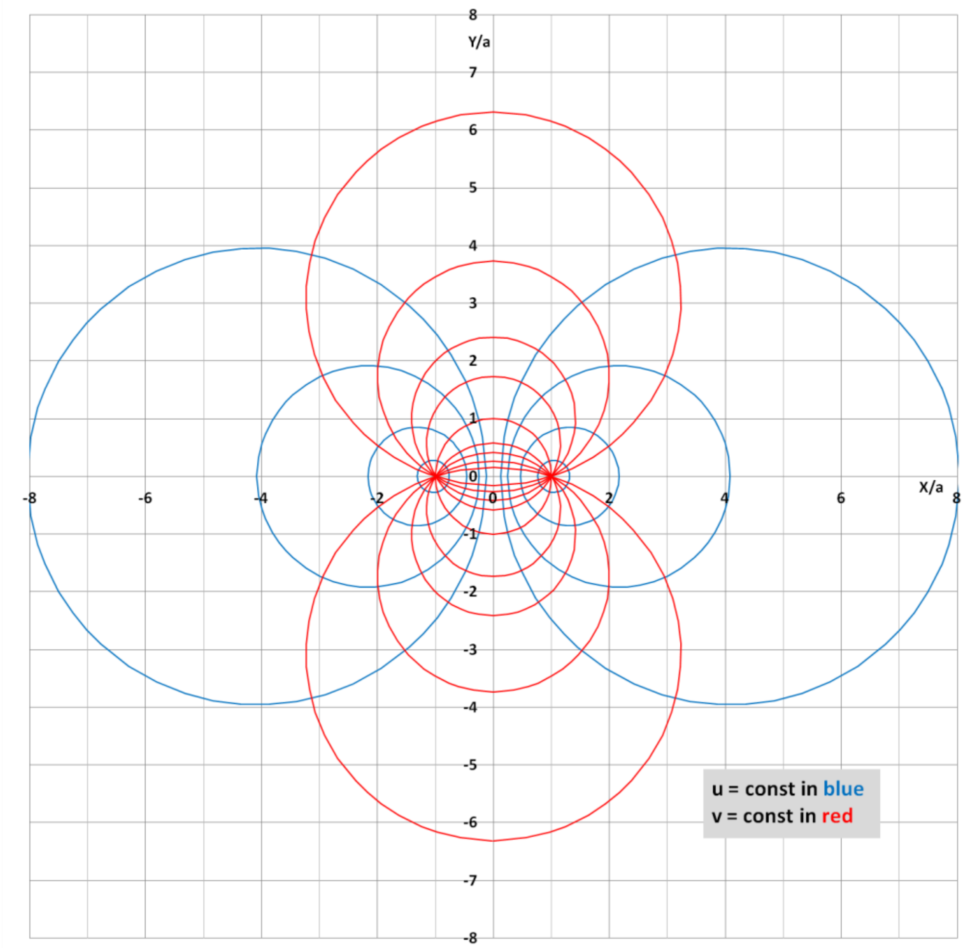

Bipolar-Cylindrical Coordinates

Bachelor thesis at the
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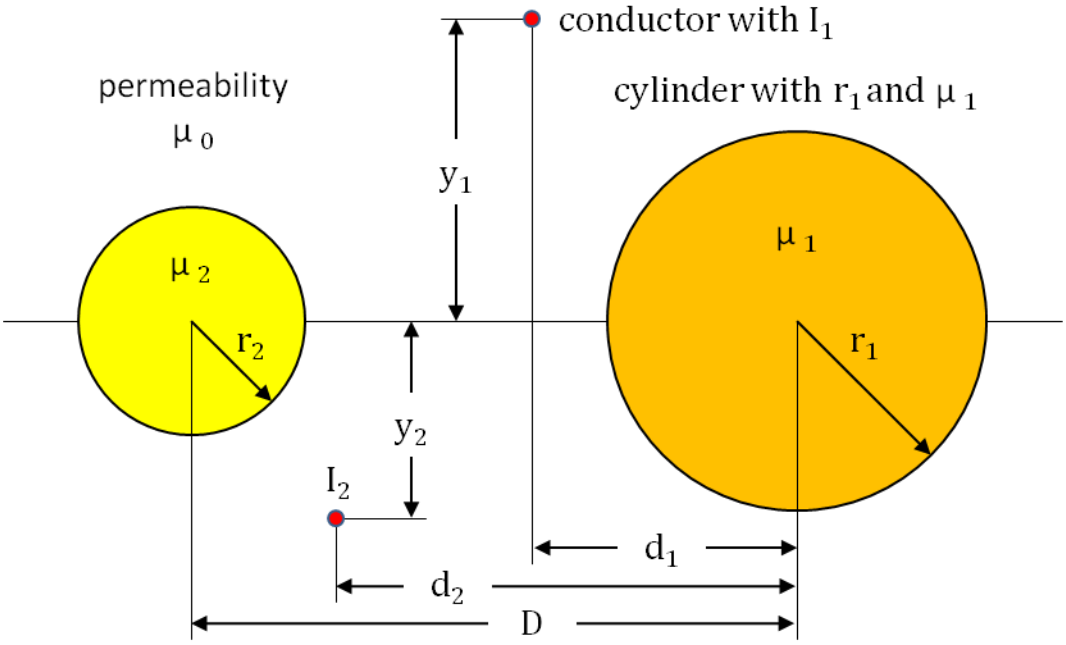
Task description:

Two cylinders with a given permeability and a certain radius r_1 and r_2 are separated in a given distance d. The permeability of both cylinders is μ_1 and μ_2.

Two live conductors with current I_1 and I_2 are located outside the cylinders.



This problem in one plane could be described by bipolar-cylindrical coordinates.

Task: Search for the mechanical force on two conductors and cylinders induced by the currents and compare them with the weight force of a typical electrical cable.

Potential solutions:

a. Determination of potential from given boundary conditions based on electro-magnetic field theory by Maxwell for the case of constant current/ field.

b. Mirroring between two partial circular areas with given permeability.

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I. Bipolar-Cylindrical Coordinates

I.1 Definition of bipolar-cylindrical coordinates

The following relationship connect a complex w plane and rectangular coordinates z

$$z = a\,\frac{e^{w}+1}{e^{w}-1} = a\,\frac{e^{w/2}+e^{-w/2}}{e^{w/2}-e^{-w/2}} = a\,coth\,\frac{w}{2}$$ equation (I.1.1)

$w = u + jv$ applying the complex w results in

$$z = \ a\,coth\,\frac{w}{2} = a\,\frac{sinh\,w}{cosh\,w-1} = a\,\frac{sinh(u+jv)}{cosh(u+jv)-1}$$ complex hyperbolic function

$$\frac{z}{a} = \frac{\sinh u \cosh jv + \cosh u \sinh jv}{\cosh u \cosh jv + \sinh u \sinh jv - 1}$$ based on cosh jv = cos v and sinh jv = j sin v

$$\frac{z}{a} = \frac{\sinh u \cos v + j \cosh u \sin v}{\cosh u \cos v + j \sinh u \sin v - 1}\ \frac{\cosh u \cos v - 1 - j \sinh u \sin v}{\cosh u \cos v - 1 - j \sinh u \sin v}$$ extended by denominator

$$\frac{z}{a} = \frac{\sinh u \cos v\,(\cosh u \cos v - 1) + \cosh u \sinh u\, sin^2\,v - j\,[sinh^2 u\, sinv \cos v - \cosh u \sin v\,(\cosh u \sin v - 1)]}{(\cosh u \cos v - 1\,)^2 + (\,sinh\,u \sin v\,)^2}$$

The denominator N will be simplified as follows

$$N = cosh^2\,u\,cos^2\,v - 2\cosh u \cos v + 1 + sinh^2\,u\,sin^2\,v$$ with $\sinh^2 u = \cosh^2 u - 1$

$$N = cosh^2\,u\,cos^2\,v - 2\cosh u \cos v + 1 + cosh^2\,u\,sin^2\,v - sin^2\,v$$ with $\cos^2 v + \sin^2 v = 1$

$$N = cosh^2\,u - 2\cosh u \cos v + cos^2\,v$$ will be

$$N = (\cosh u - \cos v)^2$$

The real part of the numerator Z will be

$$Real\,\{Z\} = \sinh u \cosh u\, cos^2\,v - \sinh u \cos v + \sinh u \cosh u\, sin^2\,v = \sinh u\,(\cosh u - \cos v)$$

The imagine part of the numerator will be

$$Im\,\{Z\} = sinh^2\,u \sin v \cos v - cosh^2\,u \sin v \cos v + \cosh u \sin v = \sin v\,(\cosh u - \cos v\,)$$

Therefore exist the following relationship between the z = x + jy plane and w = u + jv plane

$$z = x + j\,y = \frac{a \sinh u}{\cosh u - \cos v} - \ j\ \frac{a \sin v}{\cosh u - \cos v}$$ Equation (I.1.2)

I.1.1 What function describe the value u = constant within the z-plane

$\frac{x}{y} = -\frac{\sinh u}{\sin v}$ squaring

$x^2\, sin^2\, v = y^2\, sinh^2\, u = x^2\,(1 - cos^2 v)$

$x = \frac{a \sinh u}{\cosh u - \cos v}$ the real part of eq. I.1.2 will be

$x \cosh u - x \cos v = a \sinh u$ squaring

$x^2\, cos^2\, v = x^2 \cosh^2 u - 2\, a\, x \cosh u \sinh u + a^2\, sinh^2\, u$ inserting

$y^2\, sinh^2\, u \;=\; x^2 - x^2 \cosh^2 u + 2\, a\, x \cosh u \sinh u - a^2\, sinh^2\, u$

$y^2\, sinh^2\, u \;= x^2 - x^2 - x^2\, sinh^2\, u + 2\, a\, x \cosh u \sinh u - a^2\, sinh^2\, u$

$sinh^2\, u\,(\, y^2 + \, a^2 + \, x^2) = \; 2\, a\, x \cosh u \sinh u$

$x^2 + y^2 + a^2 = 2\, a\, x\, \frac{\cosh u}{\sinh u}$

$x^2 - 2\, ax\, \frac{\cosh u}{\sinh u} + \left(a \frac{\cosh u}{\sinh u}\right)^2 = \; -a^2 - y^2 + \left(a \frac{\cosh u}{\sinh u}\right)^2$

$\left(x - a \frac{\cosh u}{\sinh u}\right)^2 = \; -y^2 + a^2\, \frac{sinh^2\, u}{sinh^2\, u} + a^2\, \frac{cosh^2\, u}{sinh^2\, u}$ which will be at the end

$$\boxed{y^2 + \left(x - \frac{a \cosh u}{\sinh u}\right)^2 = \frac{a^2}{sinh^2\, u}} \qquad \text{(I.1.3)}$$

This is the equation of a circle with the general form $(x - \, x_{\,0})^2 + \, (y - y_0\,)^2 = r^2$

The values u = cont are circles on x-axis with

$(\, x - \, c_u\,)^2 + \, y^2 = (r_u)^2$	is the general circle equation	(I.1.3a)
$c_u = a\, \frac{\cosh u}{\sinh u}$	is the circle centre at x-axis	(I.1.3b)
$r_u = \left\lvert \frac{a}{\sinh u} \right\rvert$	is the circle radius	(I.1.3c)

A graph of above mentioned equation is shown on the page after next.

I.1.2 What function describe the value v = constant within the z-plane

$\frac{x}{y} = -\frac{\sinh u}{\sin v}$ squaring

$x^2\, sin^2\, v = y^2\, sinh^2\, u = y^2\,(\, cosh^2 u - 1)$

$y = -\frac{a \sin v}{\cosh u - \cos v}$ the imagine part of eq. I.1.2 will be

$y\, cosh\, u - y \cos v = -\, a \sin v \quad \gg \quad y \cosh u = y \cos v - a \sin v$ squaring

$y^2\, cosh^2\, u = y^2\, cos^2\, v - 2\, a\, y \sin v \cos v + a^2\, sin^2\, v$ inserting

$x^2\, sin^2\, v = y^2\, cos^2\, v - 2\, a\, y \sin v \cos v + a^2\, sin^2\, v - \, y^2$ with $\sin^2 v + \cos^2 v = 1$

$x^2\, sin^2\, v = \; -y^2\, sin^2\, v \; - 2\, a\, y \sin v \cos v + a^2\, sin^2\, v$ sorting

$sin^2\, v\, [\, x^2 + \, y^2 - \, a^2] = \; -2\, a\, y \sin v \cos v$ dividing by $\sin^2$ v

$y^2 + 2\, a\, y\, \frac{\cos v}{\sin v} + \left(a \frac{\cos v}{\sin v}\right)^2 \; = \; a^2 - x^2 + \left(a \frac{\cos v}{\sin v}\right)^2$ will be finally

$$x^2 + \left(y + a\, \frac{\cos v}{\sin v}\right)^2 = \left(\frac{a}{\sin v}\right)^2 \qquad \text{(I.1.4)}$$

This is also a circle equation describing with values v = const circles at the y-axis

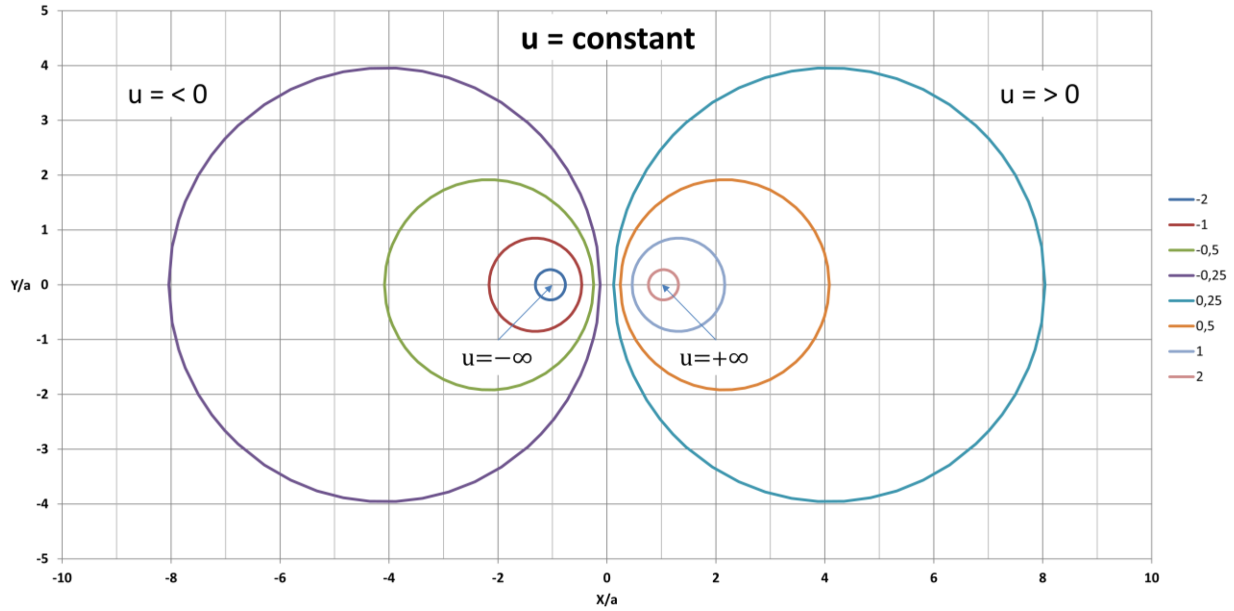
$x^2 + (\, y - \, c_v\,)^2 = (r_v)^2$ is the general circle equation (I.1.4a)

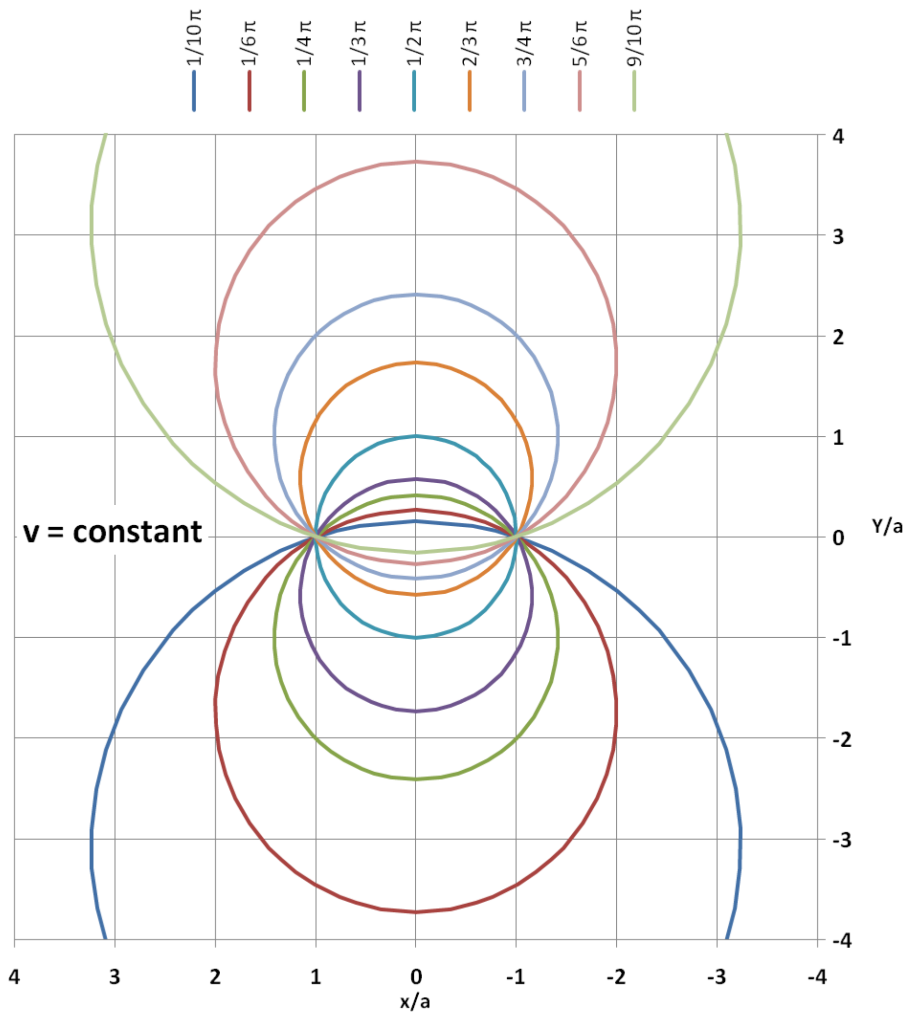
$c_v = -\, a\, \frac{\cos v}{\sin v}$ is the circle centre at y-axis (I.1.4b)

$r_v = \left|\frac{a}{\sin v}\right|$ is the circle radius (I.1.4c)

A graph of the above mentioned equation is shown on the next page.

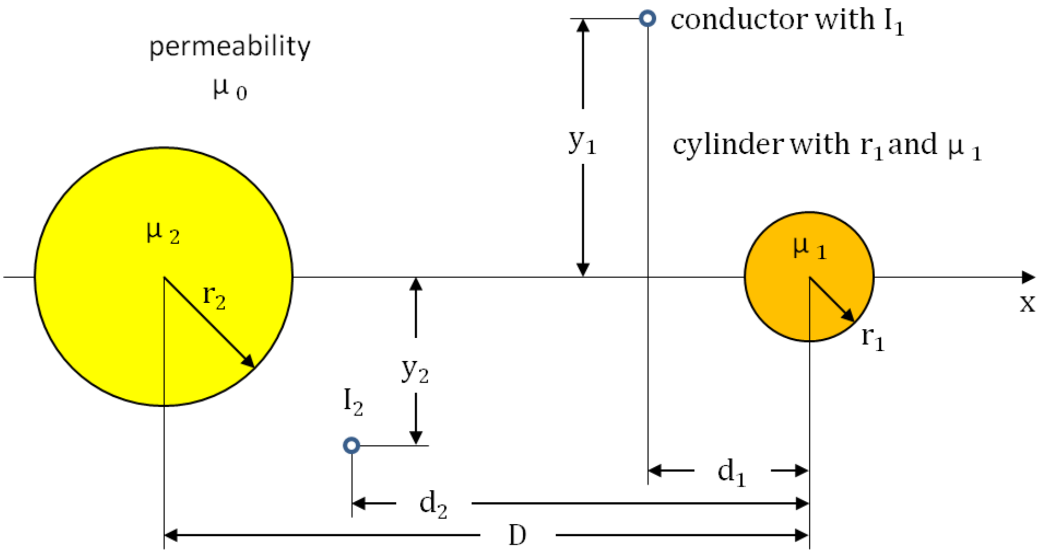
The following graph demonstrates two set of circles u and v as described in equation I.1.3/ I.1.4.





I.2 Application of bipolar-cylindrical coordinates

The following drawing described two cylinders and two life conductors



The bipolar cylindrical coordinates are defined by two sets of equations in u and v as follows:

$(x - c_u)^2 + y^2 = (r_u)^2$ with circle centre $c_u = a \frac{\cosh u}{\sinh u}$

$x^2 + (y - c_v)^2 = (r_v)^2$ with $c_v = -a \frac{\cos u}{\sin u}$

The distance D between both cylinders is the sum of both circle centres.

$$D = |c_{u_2}| + |c_{u_1}| = a \left(\left|\frac{\cosh u_1}{\text{cinh}\, u_1}\right| + \left|\frac{\cosh u_2}{\text{cinh}\, u_2}\right| \right)$$

$$\frac{D}{a} = \frac{\sqrt{1+\sinh^2 u_1}}{\sinh u_1} + \frac{\sqrt{1+\sinh^2 u_2}}{\sinh u_2}$$

The cylinder radius is defined as: $r_{u_{1,2}} = \left|\frac{a}{\sinh u_{1,2}}\right|$ > $\sinh^2 u_{1,2} = (\frac{a}{r_{1,2}})^2$

$$\frac{D}{a} = \frac{r_1}{a}\sqrt{1 + \left(\frac{a}{r_1}\right)^2} + \frac{r_2}{a}\sqrt{1 + \left(\frac{a}{r_2}\right)^2}$$ which will be finally

$$D = \sqrt{(r_1)^2 + a^2} + \sqrt{(r_2)^2 + a^2}$$

With given geometry for distance and radius of cylinders we get the reference constant a

Substitute $q^2 = (r_1)^2 + a^2$ and therefore $a^2 = q^2 - (r_1)^2$ integrate into D

$D = \sqrt{q^2} + \sqrt{(r_2)^2 + q^2 - (r_1)^2}$ $D - q = \sqrt{(r_2)^2 + q^2 - (r_1)^2}$ squaring

$(D - q)^2 = D^2 - 2Dq + q^2 = (r_2)^2 + q^2 - (r_1)^2$ q will be

$q = \frac{1}{2D} [D^2 - (r_2)^2 + (r_1)^2] = \sqrt{(r_1)^2 + a^2}$ dissolve to a results in

$$a = \sqrt{\left[\frac{D^2 - (r_2)^2 + (r_1)^2}{2\,D}\right]^2 - (r_1)^2} \qquad \text{(I.2.1)}$$

The constant a is now known, the values of $u_{,1,2}$ which describe the cylinders will be based on the cylinder radius I.1.3c as follow

$$sinh^2\, u = \frac{a^2}{(r_u)^2}$$

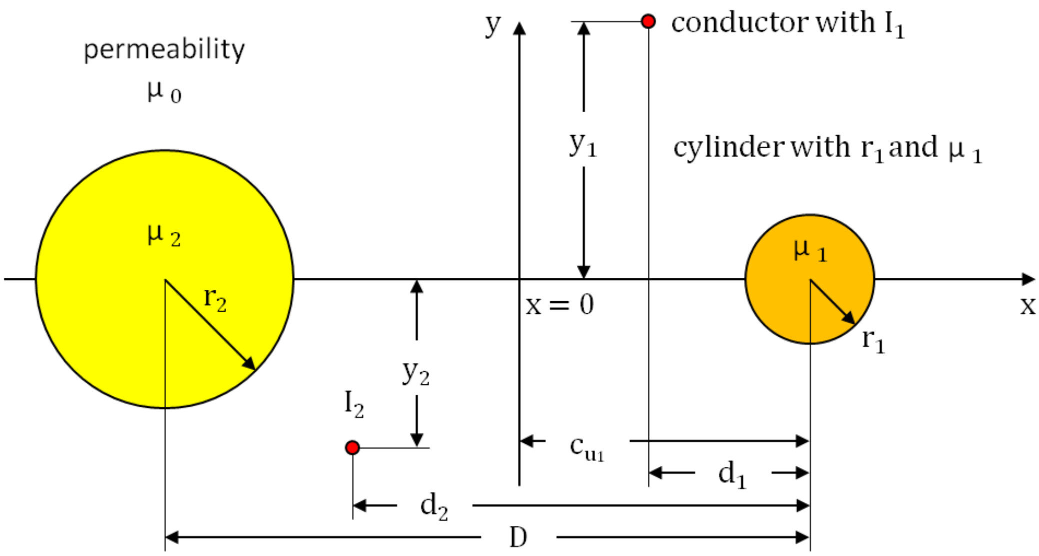
The inverse hyperbolic functions describe the circle of the cylinder in both parts of the plane.

$$u_1 = \sinh^{-1}\left(\frac{a}{r_1}\right) \quad \text{cylinder circle 1 with } u_1 = \text{constant} \qquad (I.2.2)$$

$$u_2 = -\sinh^{-1}\left(\frac{a}{r_2}\right) \quad \text{cylinder circle 2 with } u_2 = \text{constant} \qquad (I.2.3)$$

Note: the positive sign is valid for circles in the right (positive) x-plane,
the negative sign is valid for circles in the left (negative) x-plane.

The origin of coordinates on x-axis is the distance of the circle in accordance with equation I.1.3b



With this origin of coordinates we could calculate the location of the two conductors

$$x_{L_1} = c_{u_1} - d_1 \quad \text{and} \quad y_{L_1} = y_1 \quad \text{for conductor 1} \qquad (I.2.4)$$

$$x_{L_2} = c_{u_1} - d_2 \quad \text{and} \quad y_{L_2} = y_2 \quad \text{for conductor 2} \qquad (I.2.5)$$

Within the w = u + jv plane the coordinates for the conductors will be calculated with eq. I.1.3

$$y^2 + \left(x - \frac{a\cosh u}{\sinh u}\right)^2 = \frac{a^2}{sinh^2\, u}$$ squaring

$$(\,y_L)^2 + (\,x_L)^2 - \;2\,a\,x_L \coth u_L + \;a^2\,\frac{cosh^2\, u_L - 1}{sinh^2\, u_L} = 0$$ with $\sinh^2 x = \cosh^2 x - 1$

$$(\,y_L)^2 + (\,x_L)^2 + \;a^2 = \frac{2\,a\,x_L}{\tanh u_L}$$

you find an equation for the u-component u_L

Similar the calculation steps for the v-component from eq. I.1.4

$$x^2 + \left(y + a\,\frac{\cos v}{\sin v}\right)^2 = \left(\frac{a}{\sin v}\right)^2$$ squaring

$$(\,x_L)^2 + (\,y_L)^2 + \;2\,a\,y_L \cot v_L - \;a^2\,\frac{1 - cos^2\, v_L}{sin^2\, v_L} = 0$$ with $\sin^2 x = 1 - \cos^2 x$

$$(\,x_L)^2 + (\,y_L)^2 - \;a^2 = \frac{-2\,a\,y_L}{\tan v_L}$$

you find a second equation for the v-component v_L

Therefore the coordinates of the conductors within the complex bi-cylinder w = u + jv plane will be with the arcus/ inverse tangent/ tangent hyperbolic as follow:

$$u_L = \;\tanh^{-1}\frac{2\,a\,x_L}{(x_L\,)^2 + (y_L\,)^2 + a^2} \qquad \text{(I.2.6)}$$

$$v_L = \;\tan^{-1}\frac{-\,2\,a\,y_L}{(x_L\,)^2 + (y_L\,)^2 - a^2} \qquad \text{(I.2.7)}$$

I.3 Planar magneto static problem

The Maxwell equations for (non-moving) systems are:

$rot\,\vec{H} = \vec{G} + \frac{\partial \vec{D}}{\partial t}$	with H	magnetic field vector in [A/m]	Gcurrent density in [A/m^2]
$rot\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$	E	electrical field in [V/m]	
$div\,\vec{B} = 0$	B	magnetic inductive field in [Vs/m^2]	
$div\,\vec{D} = \rho$	D	electrical displacement density in [As/m^2]	
	ρ	volume charge density in [As/m^3]	

In addition we have three equations for material behaviour of field components

$\vec{B} = \mu\,\vec{H}$	with μ	di-magnetic constant/ permeability $\mu = \mu_0\,\mu_R$ in [Vs/Am]
$\vec{G} = \varkappa\,\vec{E}$	ϰ	electric conductivity in [A/Vm]
$\vec{D} = \varepsilon\,\vec{E}$	ε	di-electrical constant/ permittivity $\varepsilon = \varepsilon_0\,\varepsilon_R$ in[As/Vm]

For time constant fields the derivation of magnetic and electrical field is zero. Every source free magnetic field can be described by a new vector field $\vec{A}$ with

$\vec{B} = rot\,\vec{A}$ therefore $div\,\vec{B} = div\,rot\,\vec{A} = 0$

Based on the first Maxwell equation you will get a relationship between the current density and the new vector field A

$$\vec{G} = rot\,\vec{H} = rot\,\left(\frac{1}{\mu}\vec{B}\right) = \frac{1}{\mu}\,rot\,\vec{B} = \frac{1}{\mu}\,rot\,rot\,\vec{A}$$

$$rot\,rot\,\vec{A} = \mu\,\vec{G}$$

Applying the rules for the rotation of a rotation to a vector field yields

$$rot\,rot\,\vec{A} = grad\,div\,\vec{A} - \Delta\,\vec{A} = \mu\,\vec{G}$$ with Δ being the Laplace operator

We can assume that the sources of the new field is source free div A = 0

$$-\Delta\,\vec{A} = \mu\,\vec{G}$$

If you are using general Cartesian coordinates in u you will get the following equation for the vector potential A as a function of G with e_u being the unity vectors in $u_{1,2,3}$ direction

$$\Delta\,\vec{A} = \Delta\left(\overrightarrow{e_{u_1}}A_{u_1} + \overrightarrow{e_{u_2}}A_{u_2} + \overrightarrow{e_{u_3}}A_{u_3}\right) = -\mu\left(\overrightarrow{e_{u_1}}G_{u_1} + \overrightarrow{e_{u_2}}G_{u_2} + \overrightarrow{e_{u_3}}G_{u_3}\right) = \mu\,\vec{G} \quad \text{(I.3.1)}$$

For planar fields in u, v (independent of z-axis) we have to solve the field equation with

$$\vec{A} = \overrightarrow{e_z}\,A(u_1, u_2)$$

For an area with disappearing current density equation I.3.1 will be reduced down to

$$\Delta\,A_z = 0 \qquad \text{with} \qquad \overrightarrow{A_z} = \overrightarrow{e_z}\,A(u_1, u_2) \quad \text{(I.3.2)}$$

This is two-dimensional Laplace equation for A

$$\Delta A_z = \frac{\partial^2 A_z}{\partial {u_1}^2} + \frac{\partial^2 A_z}{\partial {u_2}^2}$$

Product approach by Bernoulli $\mathrm{A_z(u_1, u_2) = U_1(u_1)\, U_2(u_2)}$

$$\frac{\partial^2 (U_1\, U_2)}{\partial {u_1}^2} + \frac{\partial^2 (U_1\, U_2)}{\partial {u_2}^2} = U_2 \frac{\partial^2 U_1}{\partial {u_1}^2} + U_1 \frac{\partial^2 U_2}{\partial {u_2}^2} = 0$$ divide by $U_1\ U_2$

$$\frac{1}{U_1}\frac{\partial^2 U_1}{\partial {u_1}^2} + \frac{1}{U_2}\frac{\partial^2 U_2}{\partial {u_2}^2} = 0$$

Two functions which are only dependent of either u_1 or u_2 should be zero. This is only possible if both are constant equal to plus or minus p^2

$$\frac{1}{U_1}\frac{\partial^2 U_1}{\partial {u_1}^2} + \frac{1}{U_2}\frac{\partial^2 U_2}{\partial {u_2}^2} = 0$$

$\mathrm{p^2} \quad -\mathrm{p^2}$ either

$-\mathrm{q^2 q^2}$ or

Both separations lead to two usual differential equation of second order

$$\frac{\partial^2 U_1}{\partial {u_1}^2} - p^2 U_1 = 0 \quad \text{and} \quad \frac{\partial^2 U_2}{\partial {u_2}^2} + p^2 U_2 = 0 \quad \text{first possibility}$$

$$\frac{\partial^2 U_1}{\partial {u_1}^2} + q^2 U_1 = 0 \quad \text{and} \quad \frac{\partial^2 U_2}{\partial {u_2}^2} - q^2 U_2 = 0 \quad \text{second possibility}$$

Both sets of equations have the same structure and leads to same basic solutions (either in p or q).

The solutions of both differential equations are

$U_{1_p}(u_1) = A_p \cosh p u_1 + B_p \sinh p u_1$ for $p \neq 0$ and $U_{1_0}(u_1) = A_0 + B_0\, u_1$ for $p=0$

$U_{2_p}(u_2) = C_p \cos p u_2 + D_p \sin p u_2$ for $p \neq 0$ and $U_{2_0}(u_2) = C_0 + D_0\, u_2$ for $p=0$

The general solution in accordance with the product approach by Bernoulli is the sum of all potential sequence elements in p, where the separation constants in A_p, B_p, C_p and D_p has still to be evaluated.

Therefore the complete solution will be either

$$A_z(u_1, u_2) = (A_0 + B_0\, u_1)(C_0 + D_0\, u_2) + \sum_p \left(A_p \cosh p u_1 + B_p \sinh p u_1\right)\left(C_p \cos \mathrm{p} u_2 + D_p \sin p u_2\right) \tag{I.3.3}$$

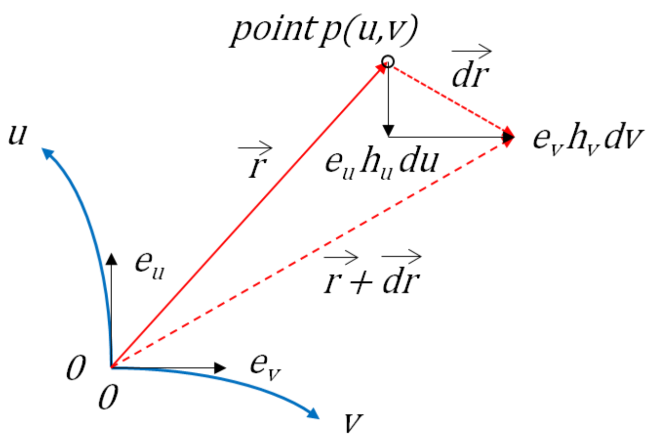
and/ or the second possibility

$$A_z(u_1, u_2) = (A_0 + B_0\, u_1)(C_0 + D_0\, u_2) + \sum_p \left(A_p \cos p u_1 + B_p \sin p u_1\right)\left(C_p \cosh p u_2 + D_p \sinh \mathrm{p} u_2\right) \tag{I.3.4}$$

Note: the separation in this form is only possible if the functions $u_1(x, y)$ and $u_2(x, y)$ have the same metrical factors $h_1 = h_2$.

I.4 General orthogonal coordinate system (u, v)

The orthogonal coordinates in u and v have the unity vectors e_u and e_v in direction of growing axis values u and v.



The orthogonal coordinates are connected with the normal Cartesian coordinates within the x-y plane via

$x = x(u,v)$
$y = y(u,v)$

The unity vectors are the tangent at point p(u,v) of the vector r in u,v

$$\vec{e_u} = \frac{1}{h_u}\frac{\partial r}{\partial u} \quad \text{with} \quad h_u = \left|\frac{\partial r}{\partial u}\right| \quad \text{being the metric factor in } h_u$$

$$\vec{e_v} = \frac{1}{h_v}\frac{\partial r}{\partial v} \quad \text{with} \quad h_v = \left|\frac{\partial r}{\partial v}\right| \quad \text{being the metric factor in } h_v$$

$$(h_u)^2 = \left(\frac{\partial r}{\partial u}\right)^2 = [e_x\frac{\partial x}{\partial u} + e_y\frac{\partial y}{\partial u}]^2 \quad \text{and} \quad (h_v)^2 = \left(\frac{\partial r}{\partial v}\right)^2 = [e_x\frac{\partial x}{\partial v} + e_y\frac{\partial y}{\partial v}]^2$$

Therefore the absolute value of metric factors

$$h_u = \sqrt{\left(\frac{\partial x}{\partial u}\right)^2 + \left(\frac{\partial y}{\partial u}\right)^2} \quad \text{and} \quad h_v = \sqrt{\left(\frac{\partial x}{\partial v}\right)^2 + \left(\frac{\partial y}{\partial v}\right)^2} \qquad \text{(I.4.1)}$$

The total differential *dr* of point vector r will be

$$\vec{dr} = \frac{\partial r}{\partial u}du + \frac{\partial r}{\partial v}dv = \vec{e_u}\,h_u\,du + \vec{e_v}\,h_v\,dv$$

Therefore the absolute value of vector line element

$$(dr)^2 = (h_u)^2(du)^2 + (h_v)^2(dv)^2$$

I.4.1 Gradient of a scalar function

The total differential of a scalar point function will be

$$d\varphi = \frac{\partial \varphi}{\partial u}du + \frac{\partial \varphi}{\partial v}dv \qquad \text{multiplying with } h_{u,v}$$

$$d\varphi = \frac{\partial \varphi}{h_u\,\partial u}h_u\,du + \frac{\partial \varphi}{h_v\,\partial v}h_v\,dv \qquad \text{resulted in}$$

$$d\varphi = \left[\vec{e_u}\frac{1}{h_u}\frac{\partial \varphi}{\partial u} + \vec{e_v}\frac{1}{h_v}\frac{\partial \varphi}{\partial v}\right][\vec{e_u}\,h_u\,du + \vec{e_v}\,h_v\,dv]$$

The first square bracket is the Gradient in general orthogonal coordinates.

$$grad\ \varphi = \vec{e_u}\frac{1}{h_u}\frac{\partial \varphi}{\partial u} + \vec{e_v}\frac{1}{h_v}\frac{\partial \varphi}{\partial v} \qquad \text{(I.4.2)}$$

I.4.2 Divergence in orthogonal coordinates

For general orthogonal coordinates (u, v, z) is the divergence of vector $\boldsymbol{V}$

$$div\,\vec{V} = \frac{1}{h_u h_u h_z}\left[\frac{\partial}{\partial u}(h_v h_z V_u) + \frac{\partial}{\partial v}(h_u h_z V_v) + \frac{\partial}{\partial z}(h_u h_v V_z)\right]$$

For the special case of a planar case with $\partial/\partial z = 0$ and metric coefficient $h_z = 1$

$$div\,\vec{V} = \frac{1}{h_u h_u}\left[\frac{\partial}{\partial u}(h_v V_u) + \frac{\partial}{\partial v}(h_u V_v)\right] \tag{I.4.3}$$

I.4.3 Divergence of gradient

For general orthogonal coordinates (u, v, z) is the divergence of vector $\boldsymbol{V}$

With V = grad φ equation I.4.3 becomes

$$div\,\vec{V} = div\ grad\ \varphi = \frac{1}{h_u h_u}\left[\frac{\partial}{\partial u}\left(h_v \frac{\partial}{h_u \partial u}\right) + \frac{\partial}{\partial v}\left(h_u \frac{\partial}{h_v \partial v}\right)\right]$$

For the special case with equal metric factors (see I.4.1) we will get

$$\Delta\varphi = \mathrm{div\ grad}\ \varphi = \frac{1}{\mathrm{h}^2}\left[\frac{\partial^2\varphi}{\partial u^2} + \frac{\partial^2\varphi}{\partial v^2}\right] \tag{I.4.4}$$

I.4.4 Rotation of vector field

The general definition for a rotation within orthogonal coordinates (u, v, z) of a vector $\boldsymbol{V}$

$$rot\,\vec{V} = \vec{e_1}(\vec{e_1}\,rot\vec{V}) + \vec{e_2}(\vec{e_2}\,rot\vec{V}) + \vec{e_3}(\vec{e_1}\,rot\vec{V})$$

with

$$\vec{e_i}\,rot\vec{V} = \frac{1}{h_j h_k}\left[\frac{\partial}{\partial u_j}(h_k\,V_k) - \frac{\partial}{\partial u_k}(h_j\,V_j)\right]$$

for all right handed coordinates systems (u_i, u_j, u_k)

For the special case of a planar system in z with $u_3 = z$ and $h_z = 1$ and $h_1 = h_2 = h$

$$\vec{e_1}\,rot\vec{V} = \frac{1}{h}\frac{\partial}{\partial u_2}(V_3) \tag{I.4.5a}$$

$$\vec{e_2}\,rot\vec{V} = -\frac{1}{h}\frac{\partial}{\partial u_1}(V_3) \tag{I.4.5b}$$

$$\vec{e_3}\,rot\vec{V} = \frac{1}{h^2}\left[\frac{\partial}{\partial u_1}(h_2 V_2) - \frac{\partial}{\partial u_2}(h_1 V_1)\right] \tag{I.4.5c}$$

I.5 Laplace equation at bipolar-cylindrical coordinates

The relationship between bipolar-cylindrical and Cartesian coordinates are defined in (I.1.2)

$$x = a\frac{\sinh u}{\cosh u - \cos v} \qquad y = -\, a\frac{\sin v}{\cosh u - \cos v}$$

The metrical factor for u-component will be

$$(h_u)^2 = \left(\frac{\partial x}{\partial u}\right)^2 + \left(\frac{\partial y}{\partial u}\right)^2 = \left[\frac{\partial}{\partial u}\left(a\frac{\sinh u}{\cosh u - \cos v}\right)\right]^2 + \left[\frac{\partial}{\partial u}\left(a\frac{-a\sin u}{\cosh u - \cos v}\right)\right]^2$$

The first square bracket in x will be

$$\frac{\partial}{\partial u}\left(a\frac{\sinh u}{\cosh u - \cos v}\right) = \frac{1}{(\cosh u - \cos v)^2}[a\cosh u\,(\cosh u - \cos v) - a\sinh u \sinh u]$$

$$= \frac{1}{(\cosh u - \cos v)^2}[a^2\cosh^2 u - a\, sinh^2 u - a\cosh u\cos v] = \frac{a(1-coshu\; cosv)}{(\cosh u - \cos v)^2} \qquad \#$$

$$\left(\frac{\partial x}{\partial u}\right)^2 = \left[\frac{1}{(\cosh u - \cos v)^2}\right]^2 (1 - 2\cosh u\cos v + cosh^2 u\; cos^2\, v)$$

The second bracket will be

$$\frac{\partial}{\partial u}\left(a\frac{-a\sin u}{\cosh u - \cos v}\right) = \frac{-a\sin v}{(\cosh u - \cos v)^2}\;[0 - \sinh u] = \frac{a\sinh u\sin v}{(\cosh u - \cos v)^2} \qquad \#\#$$

$$\left(\frac{\partial y}{\partial u}\right)^2 = \left[\frac{a}{(\cosh u - \cos v)^2}\right]^2 (\sinh^2 u\sin^2 v)$$

Therefore the metrical coefficient of h_u

$$(h_u)^2 = \left[\frac{a}{(\cosh u - \cos v)^2}\right]^2 (1 - 2\cosh u\cos v + cosh^2 u\; cos^2\, v + \;\sinh^2 u\sin^2 v)$$

$$(h_u)^2 = \left[\frac{a}{(\cosh u - \cos v)^2}\right]^2 (1 - 2\cosh u\cos v + cosh^2 u\; cos^2\, v + \;\sinh^2 u - \sinh^2 \mathrm{u}\cos^2 v)$$

$$(h_u)^2 = \left[\frac{a}{(\cosh u - \cos v)^2}\right]^2 (1 - 2\cosh u\cos v + sinh^2 u + \;cos^2\, v\,(\cosh^2 u - \sinh^2 u)$$

Note for the elements in blue $\cosh^2$-$\sinh^2$=1 and in red 1+$\sinh^2$=$\cosh^2$, therefore

$$(h_u)^2 = \left[\frac{a}{(\cosh u - \cos v)^2}\right]^2 (\cosh u - \cos v)^2 = \frac{a^2}{(\cosh u - \cos v)^2}$$

The metrical factor of the u-component is therefore simply

$$h_u = \frac{a}{\cosh u - \cos v}$$

The metrical factor for v-component will be in a similar way

$$(h_v)^2 = \left(\frac{\partial x}{\partial v}\right)^2 + \left(\frac{\partial y}{\partial v}\right)^2$$

$$\frac{\partial x}{\partial v} = \frac{a\;\sinh u}{(\cosh u - \cos v)^2}\;(0 - sinv) \qquad \#\#$$

$$\frac{\partial y}{\partial v} = \frac{1}{(\cosh u - \cos v)^2}\;[-a\cos v\,(\cosh u - \cos v) - (-\,\mathrm{a}\sin v)\sin v] = \frac{a(1-\cosh u\cos v)}{(\cosh u - \cos v)^2} \qquad \#$$

The calculation will have the same final result in v as well as in u, therefore

$h = \frac{a}{\cosh u - \cos v}$	metrical factor for bipolar-cylindrical coordinates in u, v	(I.5.1)

The Laplace equation in bipolar-cylindrical coordinates will be for the function φ(u,v)

$$\Delta\,\varphi = \frac{1}{h^2}\left[\frac{\partial^2\varphi}{\partial u^2} + \frac{\partial^2\varphi}{\partial v^2}\right] \qquad \text{with} \qquad h = \frac{a}{\cosh u - \cos v}$$

The solution of Δ φ = 0 will be for

equations (I.5.   )

u = const

$$\varphi(u,v) = (A_0 + B_0 u)(C_0 + D_0 v) + \sum_p \left(A_p \cosh pu + B_p \sinh pu\right)\left(C_p \cos pv + D_p \sin pv\right) \qquad .2$$

and/ or

v = const

$$\varphi(u,v) = (A_0 + B_0 u)(C_0 + D_0 v) + \sum_p \left(A_p \cos pu + B_p \sin pu\right)\left(C_p \cosh pv + D_p \sinh pv\right) \qquad .3$$

Note: h =h (u, v)

Important: mathematical operations grad, div or rot are dependent of metrical factor h = h (u, v)

$$grad\ \varphi = \frac{1}{\mathrm{h}}\left[\overrightarrow{e_u}\,\frac{\partial\varphi}{\partial u} + \overrightarrow{e_v}\,\frac{\partial\varphi}{\partial v}\right] \qquad \text{(I.5.4a)}$$

$$div\,\vec{V} = \frac{1}{\mathrm{h}^2}\left[\overrightarrow{e_u}\,(h\,\frac{\partial\varphi}{\partial u}) + \overrightarrow{e_v}(h\,\frac{\partial\varphi}{\partial v})\right] \qquad \text{(I.5.4b)}$$

$$\mathrm{rot}\,\vec{V} = \overrightarrow{e_u}\left\{\frac{1}{h}\,\frac{\partial V_z}{\partial v}\right\} + \overrightarrow{e_v}\left\{-\frac{1}{h}\,\frac{\partial V_z}{\partial u}\right\} + \overrightarrow{e_z}\left\{\frac{1}{\mathrm{h}^2}\left[\frac{\partial}{\partial u}\,(hV_v) - \frac{\partial}{\partial v}\,(hV_u)\right]\right\} \qquad \text{(I.5.4c)}$$

I.6 Relationship between x-y Cartesian and u-v bipolar-cylindrical coordinates

The bi-cylinder coordinates are defined by (I.1.2)

$$\vec{z} = \vec{r} = \overrightarrow{e_x}\frac{a\sinh u}{\cosh u - \cos v} + \overrightarrow{e_y}\,\frac{-a\sin v}{\cosh u - \cos v}$$

The unity vectors in bipolar-cylindrical coordinates are with the same metric factors

$$\overrightarrow{e_u} = \frac{1}{h_u}\frac{\partial r}{\partial u} = \frac{1}{h_u}\frac{\partial}{\partial u}\left\{\overrightarrow{e_x}\frac{a\sinh u}{\cosh u - \cos v} + \overrightarrow{e_y}\,\frac{-a\sin v}{\cosh u - \cos v}\right\}$$

$$= \frac{1}{h_u}\left\{\overrightarrow{e_x}\frac{a\cosh u\,(\cosh u - \cos v) - a\sinh u\sin v}{(\cosh u - \cos v)^2} + \overrightarrow{e_y}\,\frac{-a\sin v\,(-\sinh u)}{(\cosh u - \cos v)^2}\right\}$$

$$= \frac{\cosh u - \cos v}{a}\,\frac{a}{(\cosh u - \cos v)^2}\,\{\overrightarrow{e_x}(1 - \cosh u\cos v) + \overrightarrow{e_y}\,sinhu\ sinv\}$$

Therefore for unity vector in u-direction

$$\boxed{\overrightarrow{e_u} = \frac{1}{\cosh u - \cos v}\,[\overrightarrow{e_x}(1 - \cosh u\cos v) + \overrightarrow{e_y}\sinh u\sin v]} \qquad \text{(I.6.1)}$$

Same procedure for the other unity vector in v direction

$$\overrightarrow{e_v} = \frac{1}{h_v}\frac{\partial r}{\partial v} = \frac{1}{h_v}\frac{\partial}{\partial v}\left\{\overrightarrow{e_x}\frac{a\sinh u}{\cosh u - \cos v} + \overrightarrow{e_y}\,\frac{-a\sin v}{\cosh u - \cos v}\right\}$$

$$= \frac{1}{h_v}\left\{\overrightarrow{e_x}\frac{-a\ \sinh u\sin v}{(\cosh u - \cos v)^2} + \overrightarrow{e_y}\,\frac{-a\cos v\,(\cosh u - \cos v) + sinv\sin v)}{(\cosh u - \cos v)^2}\right\}$$

Therefore unity vector in v-direction

$$\boxed{\overrightarrow{e_v} = \frac{1}{\cosh u - \cos v}\,[-\overrightarrow{e_x}\sinh u\sin v + \overrightarrow{e_y}\,(1 - \cosh u\cos v)]} \qquad \text{(I.6.2)}$$

You will find the following relationship for scalar product of

$$\overrightarrow{e_u}\,\overrightarrow{e_v} = \frac{[-\sinh u\sin v + \cosh u\cos v\sinh u\sin \mathrm{v} + \sinh u\sin v - \sinh u\sin v\ \cosh u\cos v]}{(\cosh u - \cos v)^2} = 0$$

$$\overrightarrow{e_u}\,\overrightarrow{e_u} = \frac{[1 - 2\cosh u\cos v + cosh^2 u\ cos^2\,v + sinh^2 u\ sin^2\,v]}{(\cosh u - \cos v)^2}$$

$$= \frac{[1 - 2\cosh u\cos v + cosh^2 u - cosh^2\,u\ sin^2\,v + sinh^2 u\ sin^2\,v]}{(\cosh u - \cos v)^2} \qquad \mathrm{ch}^2\mathrm{x} - \mathrm{sh}^2\mathrm{x} = 1$$

$$= \frac{[1 - 2\cosh u\cos v + cosh^2 u - sin^2\,v\,]}{(\cosh u - \cos v)^2} = \frac{[cos^2\,v - 2\cosh u\cos v + cosh^2 u\,]}{(\cosh u - \cos v)^2} = 1$$

$$\overrightarrow{e_v}\,\overrightarrow{e_v} = \frac{[sinh^2\,u\ sin^2\,v + 1 - 2\cosh u\cos v + cosh^2 u - cosh^2 u\ sin^2\,v]}{(\cosh u - \cos v)^2}$$

$$= \frac{[1 - 2\cosh u\cos v + cosh^2 u - sin^2\,v\,]}{(\cosh u - \cos v)^2} = \frac{[cos^2\,v - 2\cosh u\cos v + cosh^2 u\,]}{(\cosh u - \cos v)^2} = 1$$

The u-v coordinates are working as expected/ defined.

The vector products are as follows:

$$\overrightarrow{e_u} \times \overrightarrow{e_v} = \frac{\overrightarrow{e_z}}{(\cosh u - \cos v)^2}\left[1 - 2\cosh u \cos v + cosh^2\, u\, cos^2\, v + sinh^2\, u\, sin^2\, v\right] = 0$$

$$\overrightarrow{e_u} \times \overrightarrow{e_v} = \frac{\overrightarrow{e_z}}{(\cosh u - \cos v)^2}\left[1 - 2\cosh u \cos v + cosh^2\, u\, cos^2\, v + (cosh^2 u - 1)\,(1 - cos^2 v)\right] = 0$$

$$\overrightarrow{e_u} \times \overrightarrow{e_v} = \frac{\overrightarrow{e_z}}{(\cosh u - \cos v)^2}\left[cosh^2\, u - 2\cosh u \cos v + cos^2 v\right] = \overrightarrow{e_z}$$

$$\overrightarrow{e_z} \times \overrightarrow{e_v} = \frac{1}{\cosh u - \cos v}\left[- \overrightarrow{e_x}\,(1 - \cosh u \cos v) - \overrightarrow{e_y}\, \sinh u \sin v\right] = \overrightarrow{e_u}$$

$$\overrightarrow{e_z} \times \overrightarrow{e_u} = \frac{1}{\cosh u - \cos v}\left[- \overrightarrow{e_x}\, \sinh u \sin v + \overrightarrow{e_y}\,(1 - \cosh u \cos v)\right] = \overrightarrow{e_v}$$

The bi-cylinder coordinates are creating a right handed coordinate system u > v > z

The vector $\boldsymbol{V}$ in u-v coordinates

$$\vec{V} = \overrightarrow{e_u}\, V_u + \overrightarrow{e_v}\, V_v$$

Will be transferred to the following vector components in x-y plane

$$\vec{V} = \frac{\left[\overrightarrow{e_x}(1 - \cosh u \cos v) + \overrightarrow{e_y} \sinh u \sin v\right] V_u}{\cosh u - \cos v} + \frac{\left[- \overrightarrow{e_x} \sinh u \sin v + \overrightarrow{e_y}\,(1 - \cosh u \cos v)\right] V_v}{\cosh u - \cos v}$$

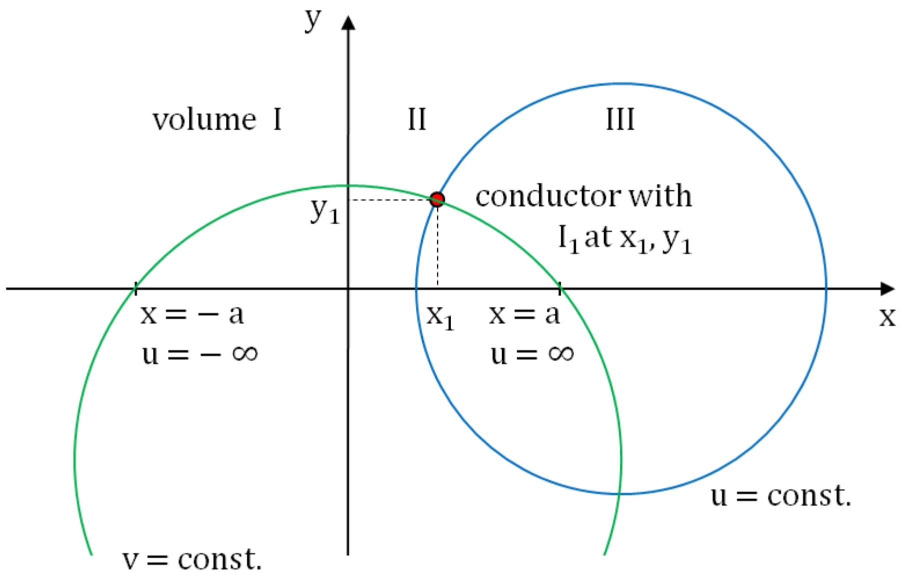
Comparison of both equations will result in components of V_x and V_y

$$V_x = \frac{(1 - \cosh u \cos v)\, V_u - \sinh u \sin v\, V_v}{\cosh u - \cos v} \tag{I.6.3}$$

$$V_y = \frac{\sinh u \sin v\;\; V_u + (1 - \cosh u \cos v)\, V_v}{\cosh u - \cos v} \tag{I.6.4}$$

II. Exciting and disturbing vector potential

II.1 Exciting vector potential of one conductor



A conductor with current I_1 is located at x_1 and y_1. The bi-cylinder coordinates within u-v plane will be in accordance with equation I.2.6/7

$$u_1 = \tanh^{-1} \frac{2\,a\,x_1}{(x_1)^2 + (x_1)^2 + a^2} \qquad\qquad v_1 = \tan^{-1} \frac{-2\,a\,y_1}{(x_1)^2 + (x_1)^2 - a^2}$$

The current flow within the z-axis will create a corresponding vector field $\boldsymbol{A}$ in z-direction

$$\vec{A} = \vec{e_z}\, A_z$$

The solution will be based on orthogonal functions in v, because on u=const. we will have boundary conditions for the cylinder in accordance with equation I.3.3

The hyperbolic function sinh(x) and cosh(x) will be changed to exponential function, because

$$\sinh x = \frac{1}{2}\,(e^{x} - e^{-x}) \qquad \text{and} \qquad \cosh x = \frac{1}{2}\,(e^{x} + e^{-x})$$

$$A_z(u,v) = (A_0 + B_0 u)(C_0 + D_0 v) + \sum_p \big(A_p \cosh pu + B_p \sinh pu\big)\big(C_p \cos pv + D_p \sin pv\big)$$

$$A_z(u,v) = (A_0 + B_0 u)(C_0 + D_0 v) + \sum_p \left(A_p \left[\frac{(e^{pu} - e^{-pu})}{2}\right] + B_p \left[\frac{(e^{pu} + e^{-pu})}{2}\right]\right)\big(C_p \cos pv + D_p \sin pv\big)$$

$$A_z(u,v) = (A_0 + B_0 u)(C_0 + D_0 v) + \sum_p \left(\frac{A_p + B_p}{2}\, e^{pu} + \frac{-A_p + B_p}{2}\, e^{-pu}\right)\big(C_p \cos pv + D_p \sin pv\big)$$

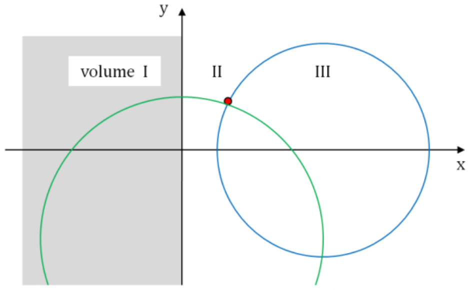
And finally renaming the round brackets with A_p, B_p to new A_p, B_p

$$\boxed{A_z(u,v) = (A_0 + B_0 u)(C_0 + D_0 v) + \sum_p \big(A_p\, e^{pu} + B_p\, e^{-pu}\big)\big(C_p \cos pv + D_p \sin pv\big)} \qquad \text{(II.1.1)}$$

The next steps will be to define the coefficient A_i, B_i, C_i and D_i based on the physical boundary conditions between the sub volumes I, II and III as well as the known current I

Volume I:	$-\infty < u < 0$	left plane until $x = 0$
Volume II:	$0 < u < u_1$	right plane between $x = 0$ and cylinder with $u = \text{const.}$
Volume III:	$u_1 < u < +\infty$	within the cylinder $u = \text{const.}$

Volume I: $-\infty < u < 0$



Due to the fact that the problem has a symmetry in rotation in the u-v plane with u = constant

$D_{0I} = 0$

The potential is limited, because the current is limited for $u = -\infty$, which means $x = -a$

$B_{0I} = 0$ and all B_{pI} has to be zero for e^{-u}

$B_{pI} = 0$

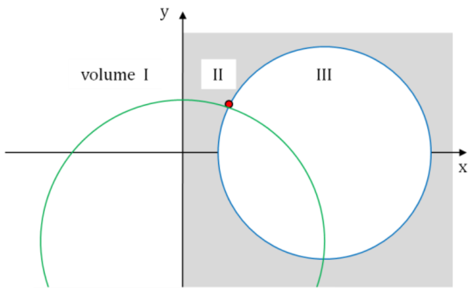
Therefore equation II.1.1 will be simplified to

$$A_{zI}(u,v) = A_{0I}C_{0I} + \sum_p (A_{pI})(C_{pI}\cos pv + D_{pI}\sin pv)\, e^{pu}$$

With renamed coefficient the potential solution will be based on

$$A_{zI}(u,v) = K_I + \sum_p (C_{pI}\cos pv + D_{pI}\sin pv)\, e^{pu}$$ for volume I (II.1.2a)

Volume II: $0 < u < u_1$



Also this case has symmetry in rotation

$D_{0II} = 0$

Due to the fact that we are in an area with limited values for u we have in this case no further possibilities to set some other coefficient in B to zero.

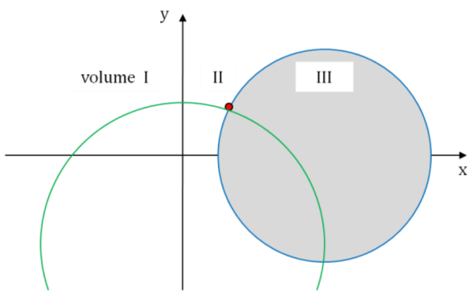
Equation II.1.1 will be simplified to

$$A_{zII} = (A_{0II}C_{0II} + B_{0II}\, C_{0II}u) + \sum_p (A_{pII}\, e^{pu} + B_{pII}\, e^{-pu})(C_{pII}\cos pv + D_{pII}\sin pv)$$

Therefore with renamed coefficient the field ansatz will be as follow.

$$A_{zII} = (K_{II} + C_{0II}u) + \sum_p (A_{pII}\, e^{pu} + B_{pII}\, e^{-pu})(C_{pII}\cos pv + D_{pII}\sin pv)$$ for volume II (II.1.2b)

Volume III: $u_1 < u < \infty$



Also this case has symmetry in rotation

$D_{0III} = 0$

The potential is limited, because the current is limited for $u = \infty$, which means $x = a$ within the cylinder

$B_{0III} = 0$

$A_{pIII} = 0$

Therefore equation II.1.1 will be

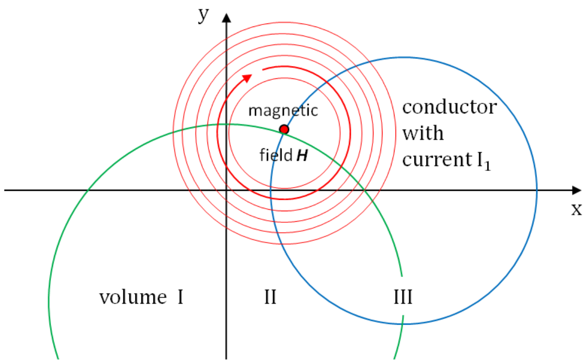
$$A_{zIII}(u,v) = A_{0III}C_{0III} + \sum_p (B_{pIII})(C_{pIII}\cos pv + D_{pIII}\sin pv)\, e^{-pu}$$ renaming

Therefore the field ansatz will be

$$A_{zIII}(u,v) = K_{III} + \sum_p (C_{pIII}\cos pv + D_{pIII}\sin pv)\, e^{-pu}$$ for volume III (II.1.2c)

The up to know unknown coefficient will be the result out of further boundary conditions of a vector field at the border between volume I and II respective volume II and III.

The field components in u-direction has to be equal at the border between two volume I and II, means



$$\vec{e_u}\, rot(\vec{A_2} - \vec{A_1}) = 0$$

$$\vec{e_u} \times rot\left(\frac{1}{\mu_2}\vec{A_2} - \frac{1}{\mu_1}\vec{A_1}\right) = \vec{K}$$

With a z-oriented field the rotation will be as described in eq. I.5.4c

$$rot\,\vec{A} = rot\,(\vec{e_z}\,A) = \frac{1}{h}\left[\vec{e_u}\frac{\partial A}{\partial v} - \vec{e_v}\frac{\partial A}{\partial u}\right]$$

$$\vec{e_u}\left\{\frac{1}{h}\left[\vec{e_u}\,\frac{\partial A_2}{\partial v} - \vec{e_v}\,\frac{\partial A_2}{\partial u}\right] - \frac{1}{h}\left[\vec{e_u}\,\frac{\partial A_1}{\partial v} - \vec{e_v}\,\frac{\partial A_1}{\partial u}\right]\right\} = 0$$

will be after multiplication

$$\boxed{\frac{\partial A_2}{\partial v} - \frac{\partial A_1}{\partial v} = 0}$$

first boundary condition for normal component of vector (II.1.3)

The boundary condition for the tangential component in the case of $\mu_1 = \mu_2 = \mu_0$

$$\vec{e_u} \times \left\{\frac{1}{\mu_0}\,\frac{1}{h}\left[\vec{e_u}\,\frac{\partial A_2}{\partial v} - \vec{e_v}\,\frac{\partial A_2}{\partial u}\right] - \frac{1}{\mu_0}\,\frac{1}{h}\left[\vec{e_u}\,\frac{\partial A_1}{\partial v} - \vec{e_v}\,\frac{\partial A_1}{\partial u}\right]\right\} = \vec{K}$$

With $\vec{e_u} \times \vec{e_u} = 0$ and $\vec{e_u} \times \vec{e_v} = \vec{e_z}$ the equation becomes

$$\boxed{-\frac{1}{\mu_0}\,\frac{1}{h}\left[\frac{\partial A_2}{\partial u} - \frac{\partial A_1}{\partial u}\right] = \vec{e_z}\,\vec{K} = \mathrm{K(v)}}$$

second boundary condition for tangent (II.1.4)

Applying the **first boundary** conditions for the normal component (in accordance with eq. II.1.3) of magnetic field at the border/ boundary between

Volume I and II – here is x = 0 at y-coordinate and therefore u = 0

$$\left(\frac{\partial A_2}{\partial v} - \frac{\partial A_1}{\partial v}\right)_{u=0} = 0$$

with eq. (II.1.2a) and (II.1.2b)

$$\left(\frac{\partial\left((K_{II}+C_{0II}u)+\sum_p\left(A_{pII}\,e^{pu}+B_{pII}\,e^{-pu}\right)\left(C_{pII}\cos pv+D_{pII}\sin pv\right)\right)}{\partial v} - \frac{\partial\left(K_I+\sum_p\left(C_{pI}\cos pv+D_{pI}\sin pv\right)e^{pu}\right)}{\partial v}\right)_{u=0} = 0$$

$$p\left[-\,C_{pI}\sin pv + D_{pI}\cos pv\right]e^{pu} = \left\{C_{0II} + p\left[A_{pII}\,e^{pu} + B_{pII}\,e^{-pu}\right]\left[-C_{pII}\sin pv + D_{pII}\cos pv\right]\right\}$$

Due to the fact that there is no linear element in volume I, the corresponding element in II will be also zero

$$\boxed{C_{0II} = 0}$$

For u = 0 the equation will be reduced to

$$\left[-\,C_{pI}\sin pv + D_{pI}\cos pv\right] = \left\{\left[A_{pII} + B_{pII}\right]\left[-C_{pII}\sin pv + D_{pII}\cos pv\right]\right\}$$

Therefore

$$C_{pI} = \left[A_{pII} + B_{pII}\right] C_{pII}$$
$$D_{pI} = \left[A_{pII} + B_{pII}\right] D_{pII}$$

Volume II and III: - the conductor has the coordinate u_1

$$\left(\frac{\partial A_3}{\partial v} - \frac{\partial A_2}{\partial v}\right)_{u=u_1} = 0$$

$$\frac{\partial}{\partial v}\left((K_{II} + C_{0II}u) + \sum_p\left(A_{pII}\,e^{pu} + B_{pII}\,e^{-pu}\right)\left(C_{pII}\cos pv + D_{pII}\sin pv\right)\right) =$$
$$\frac{\partial}{\partial v}\left(K_{III} + \sum_p\left(C_{pIII}\cos pv + D_{pIII}\sin pv\right)e^{-pu}\right) \qquad \text{for } u = u_1$$

$$p\left(A_{pII}\,e^{pu} + B_{pII}\,e^{-pu}\right)\left(-C_{pII}\sin pv + D_{pII}\cos pv\right) = p\left(-C_{pIII}\sin pv + D_{pIII}\cos pv\right)e^{-pu}$$

For u = u1 the equations for the coefficient becomes by multiplying with e^{pu}

$$\left(A_{pII}\,e^{2pu_1} + B_{pII}\right)\left(-C_{pII}\sin pv + D_{pII}\cos pv\right) = -C_{pIII}\sin pv + D_{pIII}\cos pv$$

And therefore

$$C_{pIII} = C_{pII}\left(A_{pII}\,e^{2pu_1} + B_{pII}\right)$$
$$D_{pIII} = D_{pII}\left(A_{pII}\,e^{2pu_1} + B_{pII}\right)$$

Applying the second boundary conditions for the tangent component at the border between

Volume I and II – with u=0 and v = 0

$$\left[\frac{\partial A_2}{\partial u} - \frac{\partial A_1}{\partial u}\right] = -\,\mu_0\,\mathrm{h}|_{(\mathrm{u}=0)}\mathrm{K(v)}$$

Question: why is there a current density per length unit in u direction K(v) ?

Answer: In a real physical system we have our conductor with current I placed in certain x-y coordinates, that leads to certain u-v values for the first conductor. In reality we have also to realise a backflow of the current in x and y in ∞ and this will be in u and v = 0. Otherwise we will not create a real physical system with a live conductor and a backplane (so called "earth").

With $C_{0II} = 0$

$$\frac{\partial}{\partial u}\left[\left(A_{pII}\,e^{pu} + B_{pII}\,e^{-pu}\right)\left(C_{pII}\cos pv + D_{pII}\sin pv\right)\right] - \frac{\partial}{\partial u}\left[\left(C_{pI}\cos pv + D_{pI}\sin pv\right)e^{pu}\right] = -\,\mu_0\,\mathrm{h\,K}$$

$$p\left(A_{pII}\,e^{pu} - B_{pII}\,e^{-pu}\right)\left(C_{pII}\cos pv + D_{pII}\sin pv\right) - p\left(C_{pI}\cos pv + D_{pI}\sin pv\right)e^{pu} = -\,\mu_0\,\mathrm{h\,K}$$

For u = 0 the equation will be simplified to

$$p\left(A_{pII} - B_{pII}\right)\left(C_{pII}\cos pv + D_{pII}\sin pv\right) - p\left(C_{pI}\cos pv + D_{pI}\sin pv\right) = -\,\mu_0\,\mathrm{h\,K}$$

$$\left[A_{pII}C_{pII} - B_{pII}C_{pII} - C_{pI}\right]\cos pv + \left[A_{pII}D_{pII} - B_{pII}D_{pII} - D_{pI}\right]\sin pv = -\,\frac{\mu_0}{\mathrm{p}}\,h\,\mathrm{K(v)}$$

With C_{PII} and D_{PII} we will get

$$[A_{pII}C_{pII} - B_{pII}C_{pII} - [A_{pII} + B_{pII}]\, C_{pII}]\cos pv$$

$$+ [A_{pII}D_{pII} - B_{pII}D_{pII} - [A_{pII} + B_{pII}]\, D_{pII}]\sin pv = -\frac{\mu_0}{p}\,\mathrm{h}|_{(\mathrm{u}=0)}\mathrm{K(v)}$$

$$[-2\, B_{pII}C_{pII}]\cos pv + [-2\, B_{pII}D_{pII}]\sin pv = -\frac{\mu_0}{p}\mathrm{h}|_{(\mathrm{u}=0)}\,\mathrm{K(v)}$$

$$\frac{1}{\mathrm{h}|_{(\mathrm{u}=0)}}[2\, B_{pII}C_{pII}\cos pv + 2\, B_{pII}D_{pII}\sin pv] = \frac{\mu_0}{p}\, K(v)$$

The metric factor is at u = 0

$h(u,v) = \frac{a}{\cosh u - \cos v}$ for u = 0 $h(u=0,v) = \frac{a}{1-\cos v}$ result in

$$2\,\frac{1-\cos v}{a}[B_{pII}C_{pII}\cos pv + B_{pII}D_{pII}\sin pv] = \frac{\mu_0}{p}\, K(v)$$

The current density K(v) which will be in total the current $-I_1$h(negative due to backflow !). A method of comparing a series of several elements which will be equal to one value is the orthogonal development of a function.

Note: $\int_{-\pi}^{+\pi} f(x)dx$ is equivalent to $\int_{-\pi}^{+\pi} f(v)h(v)dv$

A series development with orthogonal values of cos qv leads to the following integral equation in the boundaries between $-\pi < v < +\pi$

$$2\int_{-\pi}^{+\pi}\frac{1-\cos v}{a}\,[B_{pII}C_{pII}\cos pv + B_{pII}D_{pII}\sin pv\,]h(v)\cos qv\, dv = \frac{\mu_0}{p}\int_{-\pi}^{+\pi}\cos qv\, h(v)\, K(v)dv$$

The integrals will be

$\int_{-\pi}^{+\pi}\cos pv\cos qv\, dv = 0$ for $p \neq q$ and π for p = q

$\int_{-\pi}^{+\pi}\sin pv\cos qv\, dv = 0$ for all p, q

$\int_{-\pi}^{+\pi}\cos qv\, h(v)\, K(v)dv$ with h(u=0, v=0) $\int_{-\pi}^{+\pi}\cos 0\;\, h(v)\, K(v)dv = -I_1$

Therefore the integral will be π.

$$2\,\pi\, B_{pII}C_{pII} = -\frac{\mu_0}{p}\, I_1$$

With this step we have calculated the first coefficient, renaming to A_p.

$$\boxed{A_p = B_{pII}\, C_{pII} = -\frac{\mu_0\, I_1}{2\,\pi}\,\frac{1}{p}} \qquad \text{(II.1.5a)}$$

A similar series development with orthogonal values of sin qv leads to the following integral equation in the boundaries between $-\pi < v < +\pi$

$$2\int_{-\pi}^{+\pi}\frac{1-\cos v}{a}\,[B_{pII}C_{pII}\cos pv + B_{pII}D_{pII}\sin pv\,]h(v)\sin qv\, dv = \frac{\mu_0}{p}\int_{-\pi}^{+\pi}\sin qv\, h(v)\, K(v)dv$$

The integrals will be

$\int_{-\pi}^{+\pi} \cos pv \sin qv \, dv = 0$ for all p, q

$\int_{-\pi}^{+\pi} \sin pv \sin qv \, dv = 0$ for $p \neq q$ and π for $p = q$

$\int_{-\pi}^{+\pi} \sin qv \, h(v)\, K(v) dv$ with h(u=0, v=0) $\int_{-\pi}^{+\pi} \sin 0 \; h(v)\, K(v) dv = 0$

The first integral in cos(pv) sin(qv) is zero, the second will be constant π. The right part of the equation will be for v = 0 also zero and therefore the coefficient could be set to

$$2\, \pi\, B_{pII} D_{pII} = 0$$

With this step we have calculated the second coefficient as follows:

$$\boxed{B_{pII}\, D_{pII} = 0}$$

A similar calculation will be applied for the second boundary conditions for the tangent component at the border between

Volume II and III – with $u=u_1$

$$\left[\frac{\partial A_3}{\partial u} - \frac{\partial A_2}{\partial u}\right] = -\mu_0 \, \mathrm{h}|_{(\mathrm{u}=\mathrm{u}_1)} \mathrm{K(v)}$$

Using the field ansatz for field in volume III respective II lead to (with $C_{0II} = 0$), insert the already defined values for C_{pIII} and D_{pIII} (see page 21)

$$\left\{\frac{\partial}{\partial u}\left[K_{III} + \sum_p \left(C_{pIII} \cos pv + D_{pIII} \sin pv\right) e^{-pu}\right] - \frac{\partial}{\partial u}\,[\ldots]\right\} = -\mu_0 \, \mathrm{h}|_{(\mathrm{u}=\mathrm{u}_1)} \mathrm{K(v)}$$

$$\left\{\ldots - \frac{\partial}{\partial u}\,\left[(K_{II} + C_{0II} u) + \sum_p \left(A_{pII}\, e^{pu} + B_{pII}\, e^{-pu}\right)\left(C_{pII} \cos pv + D_{pII} \sin pv\right)\right]\right\} = -\mu_0 \, \mathrm{h}|_{(\mathrm{u}=\mathrm{u}_1)} \mathrm{K(v)}$$

Differentiation lead to

$$-p\,\left(C_{pII}(A_{pII}\, e^{2pu_1} + B_{pII}\,) \cos pv + D_{pII}(A_{pII}\, e^{2pu_1} + B_{pII}\,) \sin pv\right) e^{-pu} - \ \ldots$$

$$\ldots p\left[\left(A_{pII}\, e^{pu} - B_{pII}\, e^{-pu}\right)\left(C_{pII} \cos pv + D_{pII} \sin pv\right)\right] = -\mu_0 \, \mathrm{h} \, \mathrm{K(v)}$$

For $u = u_1$ we have to solve

$$-p\,\left\{\left[C_{pII} B_{pII} + C_{pII} A_{pII}\, e^{2pu_1}\,\right] e^{-pu_1} + \left[A_{pII} C_{pII} e^{+pu_1} - B_{pII} C_{pII} e^{-pu_1}\,\right]\right\} \cos pv - \cdots$$

$$-p\,\left\{\left[D_{pII} B_{pII} + D_{pII} A_{pII}\, e^{2pu_1}\,\right] e^{-pu_1} + \left[A_{pII} D_{pII} e^{+pu_1} - B_{pII} D_{pII} e^{-pu_1}\,\right]\right\} \sin pv = -\mu_0 \, \mathrm{h}|_{(\mathrm{u}=\mathrm{u}_1)} \mathrm{K(v)}$$

With $B_{pII} D_{pII} = 0$ the equation will be reduced to

$$2\, C_{pII} A_{pII}\, e^{pu_1} \cos pv + 2\, D_{pII} A_{pII}\, e^{pu_1} \sin pv = +\frac{\mu_0}{p}\, \mathrm{h}|_{(\mathrm{u}=\mathrm{u}_1)} \mathrm{K(v)}$$

As before a series development to orthogonal elements in cos qv with $h(u_1, v)$

$$2 \int_{-\pi}^{+\pi} C_{pII} A_{pII}\; e^{pu_1} \cos pv \cos qv\; h\; dv + 2 \int_{-\pi}^{+\pi} D_{pII} A_{pII} e^{pu_1} \sin pv \cos qv\; h\; dv = \int_{-\pi}^{+\pi} \frac{\mu_0}{p}\, \mathrm{h}|_{(\mathrm{u}=\mathrm{u}_1)} \mathrm{K(v)} \cos qv\; h\; dv$$

The metric factor h(u,v) is nearly constant at the location of the conductor, therefore with the current I the integral on the right side will be for $v = v_1$ (the location of conductor) as before.

The integrals of cos(pv) cos(qv) will be π for p=q, else zero, and cos(pv) sin(pv) will be always zero. Therefore for v = v_1

$$2\, C_{pII} A_{pII}\, e^{pu_1} \pi\, h(u_1, v_1) = \frac{\mu_0}{p}\, \mathrm{h}|_{(\mathrm{u}=\mathrm{u}_1)} \int_{-\pi}^{+\pi} \mathrm{K(v)} \cos qv\; h\, dv = \frac{\mu_0}{p}\, \mathrm{h}|_{(\mathrm{u}_1,\mathrm{v}_1)} \cos \mathrm{pv}_1\; \mathrm{I}_1$$

Now we have the second coefficient calculated for v = v_1 and rename this constant to B_p

$$\boxed{B_p = C_{pII} A_{pII} = \frac{\mu_0 I_1}{2\pi} \frac{\cos pv_1}{p e^{pu_1}}} \qquad \text{(II.1.5b)}$$

Again the similar procedure for the orthogonal development with sin qv

$$2 \int_{-\pi}^{+\pi} C_{pII} A_{pII}\; e^{pu_1} \cos pv \sin qv\; h\, dv + 2 \int_{-\pi}^{+\pi} D_{pII} A_{pII} e^{pu_1} \sin pv \sin qv\; h\, dv = \int_{-\pi}^{+\pi} \frac{\mu_0}{p}\, \mathrm{h}|_{(\mathrm{u}=\mathrm{u}_1)} \mathrm{K(v)} \sin qv\; h\, dv$$

First integral cos pv sin qv will be zero

$$2\, D_{pII} A_{pII}\, e^{pu_1} \pi\, h(u_1, v_1) = \frac{\mu_0}{p}\, \mathrm{h}|_{(\mathrm{u}=\mathrm{u}_1)} \int_{-\pi}^{+\pi} \mathrm{K(v)} \sin qv\; h\, dv = \frac{\mu_0}{p}\, \mathrm{h}|_{(\mathrm{u}_1,\mathrm{v}_1)} \sin \mathrm{pv}_1\, \mathrm{I}_1$$

Finally as before - and renaming

$$\boxed{C_p = D_{pII} A_{pII} = \frac{\mu_0 I_1}{2\pi} \frac{\sin pv_1}{p e^{pu_1}}} \qquad \text{(II.1.5c)}$$

We have now all constant defined as follows:

$$A_p = B_{pII}\, C_{pII} \qquad B_p = C_{pII} A_{pII} \qquad C_p = D_{pII} A_{pII}$$

The coefficients in all three volumes are as follows

$$C_{pI} = \left[A_{pII} + B_{pII}\right] C_{pII} = B_p + A_p$$

$$D_{pI} = \left[A_{pII} + B_{pII}\right] D_{pII} = C_p \qquad \text{due to } B_{pII}\, D_{pII} = 0$$

$$A_{pII} C_{pII} = B_p \qquad A_{pII} D_{pII} = C_p \qquad \mathrm{B_{pII} C_{pII}} = \mathrm{A_p} \qquad \mathrm{B_{pII}\, D_{pII}} = 0$$

$$C_{pIII} = C_{pII}(A_{pII}\, e^{2pu_1} + B_{pII}) = B_p\, e^{2pu_1} + A_p$$

$$D_{pIII} = D_{pII}(A_{pII}\, e^{2pu_1} + B_{pII}) = C_p\, e^{2pu_1} \qquad \text{due to } \mathrm{B_{pII}\, D_{pII}} = 0$$

The exciting vector potential of a conductor is described as follows within the three volumes

Volume I: $-\infty < u < 0$

$$A_{zI}(u, v) = K_I + \sum_{p=1}^{\infty} \left[(A_p + B_p) \cos pv + C_p \sin pv \right] e^{pu} \qquad \text{(II.1.6a)}$$

Volume II: $0 < u < u_1$

$$A_{zII} = K_{II} + \sum_{p=1}^{\infty} \left[(B_p\, e^{pu} + A_p e^{-pu}) \cos pv + C_p\, e^{pu} \sin pv \right] \qquad \text{(II.1.6b)}$$

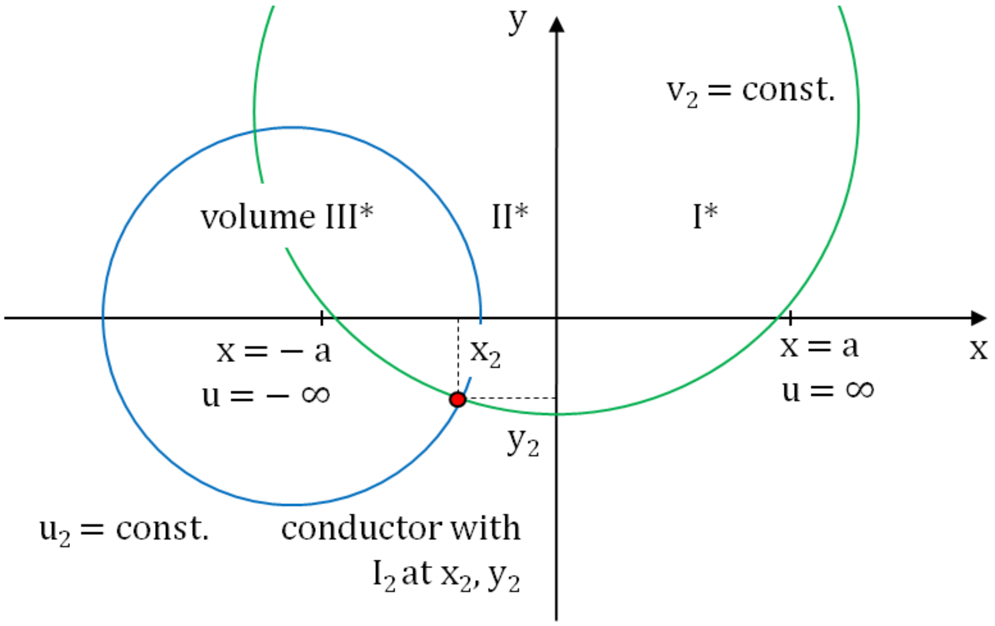
Volume III: $u_1 < u < \infty$

$$A_{zIII} = K_{III} + \sum_{p=1}^{\infty} \left[(A_p + B_p e^{2pu_1}) \cos pv + C_p e^{2pu_1} \sin pv \right] e^{-pu} \qquad \text{(II.1.6c)}$$

The basic coefficient A_p, B_p and C_p see equations (II.1.5 a - c)

II.2 Exciting vector potential of a second conductor

For a second conductor with current I_2 at x_2; y_2 – respective u_2, v_2 the solution will be similar if the second conductor is located at $x < 0$ (in the left region). The solution will be based on the already known solution for a conductor in the right plane, $x > 0$ just by mirroring and replacing u with –u.



The basic coefficients of the solutions and the equations are:

$$A_p^* = -\frac{\mu_0\, I_2}{2\,\pi}\,\frac{1}{p} \qquad B_p^* = \frac{\mu_0 I_2}{2\pi}\,\frac{\cos p v_2}{p e^{-p u_2}} \qquad C_p^* = \frac{\mu_0 I_2}{2\pi}\,\frac{\sin p v_2}{p e^{-p u_2}} \tag{II.2.1}$$

Volume III*: $-\infty < u < u_2$

$$A_{zIII}^* = K_{III}^* + \sum_{p=1}^{\infty}\left[\left(A_p^* + B_p^* e^{-2pu_2}\right)\cos pv + C_p^* e^{-2pu_2}\,\sin pv\right]e^{+pu} \tag{II.2.2a}$$

Volume II*: $u_2 < u < 0$

$$A_{zII}^* = K_{II}^* + \sum_{p=1}^{\infty}\left[\left(B_p^*\, e^{-pu} + A_p^* e^{+pu}\right)\cos pv + C_p^*\, e^{-pu}\sin pv\right] \tag{II.2.2b}$$

Volume I*: $0 < u < \infty$

$$A_{zI}^* = K_I^* + \ + \sum_{p=1}^{\infty}\left[\left(A_p^* + B_p^*\right)\cos pv + C_p^* \sin pv\right] e^{-pu} \tag{II.2.2c}$$

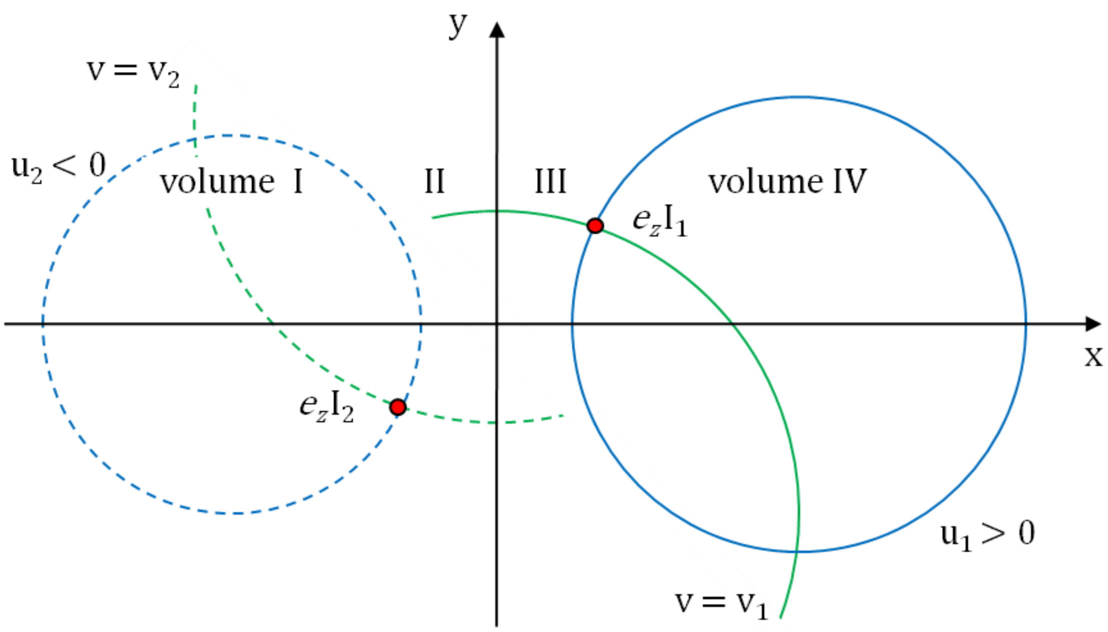
II.3 Exciting vector potential of two conductors

Two conductors with current I_1 and I_2 could be located in three combinations within the given volume:

a) Both conductors are located within positive x-values – ($u_{1,2} > 0$)
b) Both conductors are located within negative x-values – ($u_{1,2} < 0$)
c) One conductor is within the positive, the other within the negative plane – ($u_2 < 0$, $u_1 > 0$)

Each combination requires a different set of equations (which are off course based on the already known solution given in the previous chapters.

We will analyse case with two conductors in the positive and negative plane as detailed in the following drawing.



With the conductor coordinates in u, v coordinates (see eq. I.2.6/7 and $r_{1,2}$ are the cylinder radius of permeable cylinder with material coefficient μ_1, μ_2 and D the distance between the cylinder

$$u_L = \tanh^{-1} \frac{2\,a\,x_L}{(x_L)^2 + (x_L)^2 + a^2} \qquad v_L = \tan^{-1} \frac{-2\,a\,y_L}{(x_L)^2 + (x_L)^2 - a^2}$$

$$a = \sqrt{\left[\frac{D^2 - (r_2)^2 + (r_1)^2}{2\,D}\right]^2 - (r_1)^2}$$ coordinate constant eq. I.2.1

The total exciting field potential is based on the given solutions in previous chapter as follow:

$-\infty < u < u_2$	$A_{eI} = A_I + A_{III}^*$
$u_2 < u < 0$	$A_{eII} = A_I + A_{II}^*$
$0 < u < u_1$	$A_{eIII} = A_{II} + A_I^*$
$u_1 < u < \infty$	$A_{eIV} = A_{III} + A_I^*$

We will have the following general solution within the different partial volumes I … IV

Volume I $\quad -\infty < u < u_2$

$$A_{eI} = A_I + A_{III}^* \quad = K_I + \sum_{p=1}^{\infty}\left[\left(A_p + B_p\right)\cos pv + C_p \sin pv\right] e^{pu} + \ \dots$$

$$+ K_{III}^* + \sum_{p=1}^{\infty}\left[\left(A_p^* + B_p^* e^{-2pu_2}\right)\cos pv + C_p^* e^{-2pu_2} \sin pv\right] e^{+pu}.$$

$$A_{eI} = \underbrace{K_I + K_{III}^*}_{K_{eI}} + \sum_{p=1}^{\infty}\left[\underbrace{\left(A_p + B_p + A_p^* + B_p^* e^{-2pu_2}\right)}_{D_{Ip}}\cos pv + \underbrace{\left(C_p + C_p^* e^{-2pu_2}\right)}_{E_{Ip}} \sin pv\right] e^{pu}$$

Volume II $\quad u_2 < u < 0$

$$A_{eII} = A_I + A_{II}^* \quad = K_I + \sum_{p=1}^{\infty}\left[\left(A_p + B_p\right)\cos pv + C_p \sin pv\right] e^{pu} + \ \dots$$

$$+ K_{II}^* + \sum_{p=1}^{\infty}\left[\left(B_p^* e^{-pu} + A_p^* e^{+pu}\right)\cos pv + C_p^* e^{-pu} \sin pv\right]$$

$$A_{eII} = \underbrace{K_I + K_{II}^*}_{K_{eII}} + \sum_{p=1}^{\infty}\left[\underbrace{\left(A_p + B_p + A_p^*\right)}_{F_{IIp}}\cos pv + C_p \sin pv\right] e^{pu} + \ \dots$$

$$+ \sum_{p=1}^{\infty}\left[B_p^* \cos pv + C_p^* \sin pv\right] e^{-pu}$$

Volume III $\quad 0 < u < u_1$

$$A_{eIII} = A_{II} + A_I^* \quad = K_{II} + \sum_{p=1}^{\infty}\left[\left(B_p e^{pu} + A_p e^{-pu}\right)\cos pv + C_p e^{pu} \sin pv\right] + \dots$$

$$+ K_I^* + \sum_{p=1}^{\infty}\left[\left(A_p^* + B_p^*\right)\cos pv + C_p^* \sin pv\right] e^{-pu}.$$

$$A_{eIII} = \underbrace{K_{II} + K_I^*}_{K_{eIII}} + \sum_{p=1}^{\infty}\left[\underbrace{\left(A_p + A_p^* + B_p^*\right)}_{F_{IIIp}}\cos pv + C_p^* \sin pv\right] e^{-pu} + \ \dots$$

$$+ \sum_{p=1}^{\infty}\left[B_p \cos pv + C_p \sin pv\right] e^{pu}$$

Volume IV $\quad u_1 < u < \infty$

$$A_{eIV} = A_{III} + A_I^* \quad = K_{III} + \sum_{p=1}^{\infty}\left[\left(A_p + B_p e^{2pu_1}\right)\cos pv + C_p e^{2pu_1} \sin pv\right] e^{-pu} + \ \dots$$

$$+ K_I^* + \ + \sum_{p=1}^{\infty}\left[\left(A_p^* + B_p^*\right)\cos pv + C_p^* \sin pv\right] e^{-pu}$$

$$A_{eIV} = \underbrace{K_{III} + K_I^*}_{K_{eIV}} + \sum_{p=1}^{\infty}\left[\underbrace{\left(A_p + B_p e^{2pu_1} + A_p^* + B_p^*\right)}_{D_{IVp}}\cos pv + \underbrace{\left(C_p e^{2pu_1} + C_p^*\right)}_{E_{IVp}} \sin pv\right] e^{-pu}$$

With these new constant we will have the following final solution for the magnetic potential of two conductors for the case $u_1 > 0$ and $u_2 < 0$

Volume I $\quad -\infty < u < u_2$ (II.3.1)

$$A_{eI} = K_{eI} + \sum_{p=1}^{\infty}\left[D_{Ip}\cos pv + E_{Ip}\sin pv\right] e^{pu}$$

Volume II $\quad u_2 < u < 0$ (II.3.2)

$$A_{eII} = K_{eII} + \sum_{p=1}^{\infty}\left[\left(F_{IIp}\cos pv + C_p \sin pv\right) e^{pu} + \left(B_p^* \cos pv + C_p^* \sin pv\right)e^{-pu}\right]$$

Volume III $\quad 0 < u < u_1$ (II.3.3)

$$A_{eIII} = K_{eIII} + \sum_{p=1}^{\infty}\left[\left(F_{IIIp}\cos pv + C_p^* \sin pv\right)e^{-pu} + \left(B_p \cos pv + C_p \sin pv\right)e^{pu}\right]$$

Volume IV $\quad u_1 < u < \infty$ (II.3.4)

$$A_{eIV} = K_{eIV} + \sum_{p=1}^{\infty}\left[D_{IVp}\cos pv + E_{IVp}\sin pv\right]e^{-pu}$$

With the following short cuts

$$D_{Ip} = A_p + B_p + A_p^* + B_p^* e^{-2pu_2} \qquad D_{IVp} = A_p + B_p e^{2pu_1} + A_p^* + B_p^* \qquad \text{(II.3.5a,b)}$$

$$E_{Ip} = C_p + C_p^* e^{-2pu_2} \qquad E_{IVp} = C_p e^{2pu_1} + C_p^* \qquad \text{(II.3.6a,b)}$$

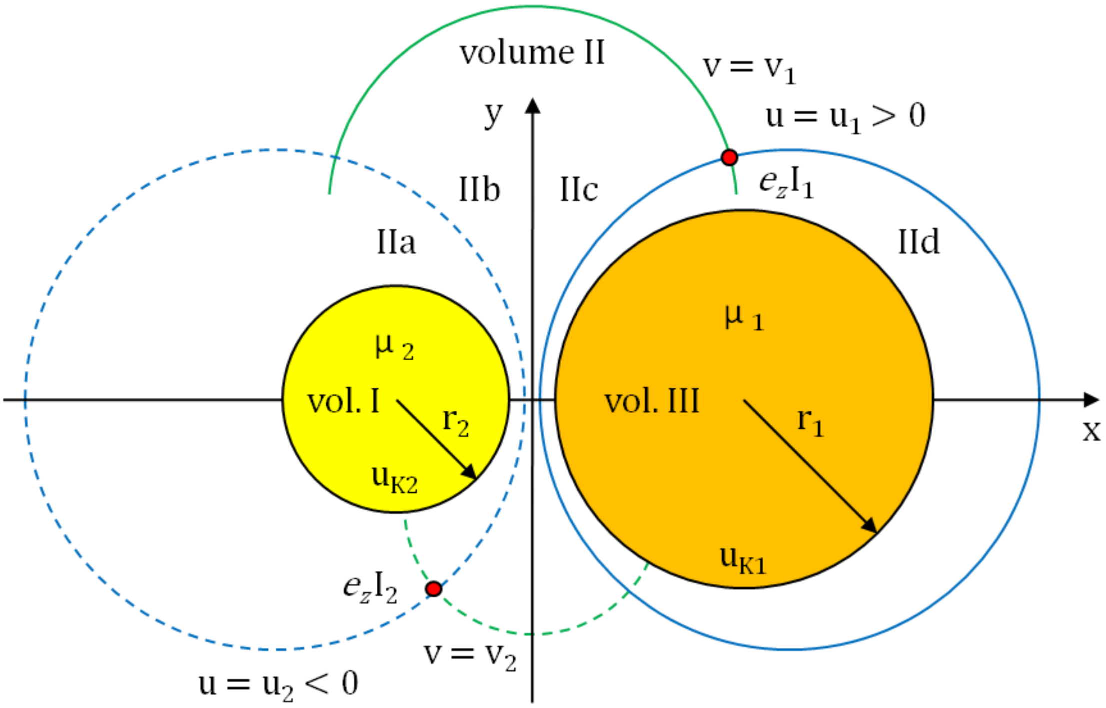
$$F_{IIp} = A_p + B_p + A_p^* \qquad F_{IIIp} = A_p + A_p^* + B_p^* \qquad \text{(II.3.7a,b)}$$

The coefficients A_p, B_p and C_p are defined in previous chapters – see equation II.1.5 & II.2.1

The four constants K_{eI} till K_{eIV} describe a constant background potential, not created by our current I_1 and I_2. There is no physical condition for defining these constant – they could without any problem set to zero (or any other value).

II.4 Vector potential of two conductors between two cylinders

Within the x-y plane or u-v plane we have two cylinders with a certain permeability μ_1 and μ_2 and two exciting conductors with current I_1 and I_2. The current induced an exciting potential A_e which will create a magnetic field.



The following potential ansatz is valid for the disturbing "Stör" potential and is based on the exciting potential/ field ansatz with a general combination of trigonometric and exponential functions as used in the previous equations.

Volume I $\quad -\infty < u < u_{K2}$ where u_{K2} is the boundary of cylinder II

$$A_{IStör} = \sum_{p=1}^{\infty}\left[A_{pStör}\cos pv + B_{pStör}\sin pv\right] e^{pu} \tag{II.4.1}$$

Volume II $\quad u_{K2} < u < u_{K1}$

$$A_{IIStör} = \left\{\begin{matrix} A_{eI} \\ A_{eII} \\ A_{eIII} \\ A_{eIV} \end{matrix}\right\} + \sum_{p=1}^{\infty}\left[C_{pStör}\cos pv + D_{pStör}\sin pv\right]\left[E_{pStör}e^{pu} + F_{pStör}e^{-pu}\right] \tag{II.4.2}$$

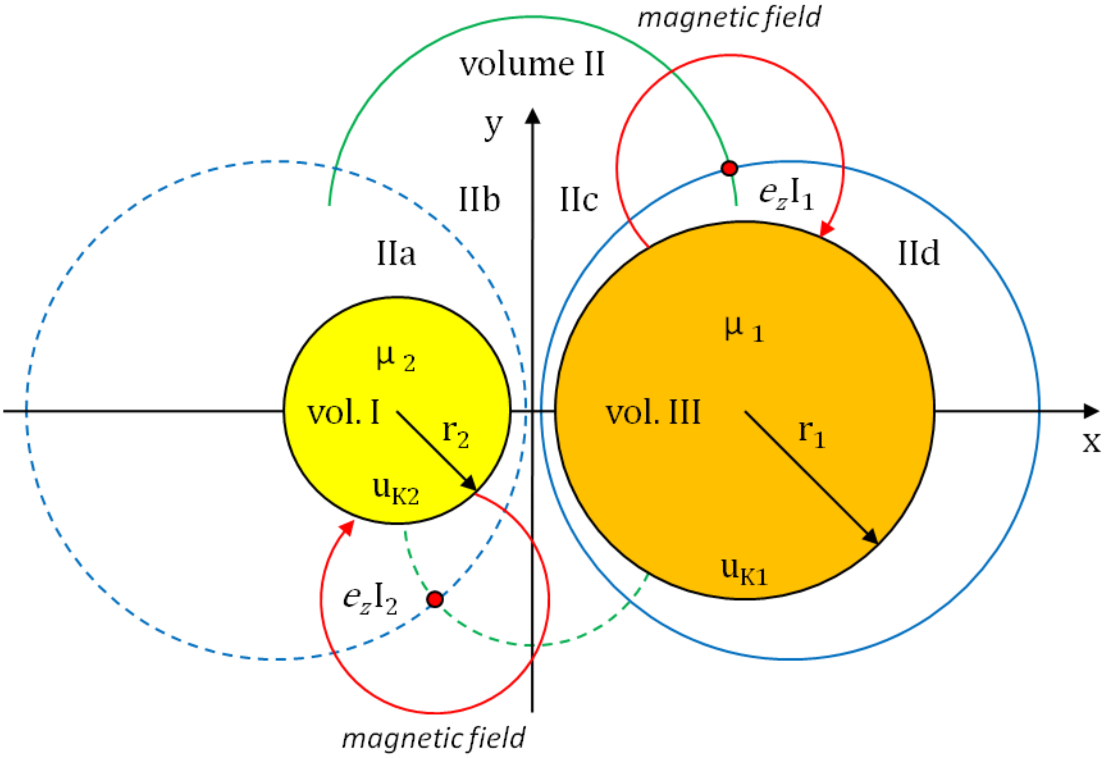
Volume III $\quad u_{K1} < u < \infty$ where u_{K1} is the boundary of cylinder I

$$A_{IIIStör} = \sum_{p=1}^{\infty}\left[G_{pStör}\cos pv + H_{pStör}\sin pv\right] e^{-pu} \tag{II.4.3}$$

Note: The exciting potential $A_{eI\,...\,IV}$ is depending of the selected volume, based on solutions presented in previous sections.

The eight "Stör" coefficients have to be defined at the various boundaries for the normal and tangential component of the magnetic field.

Similar as described in section II.1 we have two conditions for the normal and tangent component of the magnetic field.



First condition:

Normal component is equal at the cylinder boundary with u = $u_{K1,2}$

$$\vec{e_u}\, rot(\vec{A_1} - \vec{A_2})$$

$$\left(\frac{\partial A_1}{\partial v} - \frac{\partial A_2}{\partial v}\right)_{u_K} = 0 \qquad \text{(II.4.4)}$$

Second condition:

Tangent component is jumping in accordance with the current density at the cylinder with u = $u_{K1,2}$ which is in our case not the true, because we have defined that the conductors are located in volume II.

$$\vec{e_u} \times rot\left(\frac{1}{\mu_2}\vec{A_2} - \frac{1}{\mu_1}\vec{A_1}\right) = \vec{K} = 0$$

$$\left(\frac{1}{\mu_2}\frac{\partial A_2}{\partial u} - \frac{1}{\mu_1}\frac{\partial A_1}{\partial u}\right)_{u_K} = 0 \qquad \text{(II.4.5)}$$

With these conditions at u = $u_{K1,2}$ we have in total 2 x 2 for sinus and cosines function, in total 8 equations for the eight “Stör” components.

First condition for normal component

for $u = u_{K2}$

$$\left(\frac{\partial A_1}{\partial v} - \frac{\partial A_2}{\partial v}\right)_{u_{K2}} = 0 = \left[\frac{\partial A_{IStör}}{\partial v} - \left(\frac{\partial A_{eI}}{\partial v} + \frac{\partial A_{IIStör}}{\partial v}\right)\right]_{u_{K2}}$$

$$0 = \left[\frac{\partial}{\partial v}\left([A_{pStör}\cos pv + B_{pStör}\sin pv]e^{pu}\right) - \left(\frac{\partial}{\partial v}\left([D_{Ip}\cos pv + E_{Ip}\sin pv]e^{pu}\right) + \frac{\partial}{\partial v}\left([C_{pStör}\cos pv + D_{pStör}\sin pv][E_{pStör}e^{pu} + F_{pStör}e^{-pu}]\right)\right)\right]_{u_{K2}}$$

$$p[-A_{pStör}\sin pv + B_{pStör}\cos pv]e^{pu} - p[-D_{Ip}\sin pv + E_{Ip}\cos pv]e^{pu} + \ldots$$
$$-p[-C_{pStör}\sin pv + D_{pStör}\cos pv][E_{pStör}e^{pu} + F_{pStör}e^{-pu}] = 0$$

For $u = u_{K2}$ and sorting to sinus and cosines, dividing by p result in

$$\sin pv\ \{-A_{pStör}e^{pu_{K2}} + D_{Ip}e^{pu_{K2}} + C_{pStör}[E_{pStör}e^{pu_{K2}} + F_{pStör}e^{-pu_{K2}}]\} = 0$$

$$\cos pv\ \{B_{pStör}e^{pu_{K2}} - E_{Ip}e^{pu_{K2}} - D_{pStör}[E_{pStör}e^{pu_{K2}} + F_{pStör}e^{-pu_{K2}}]\} = 0$$

We have now the first two equations for the coefficient.

$$A_{pStör}e^{pu_{K2}} - D_{Ip}e^{pu_{K2}} - C_{pStör}E_{pStör}e^{pu_{K2}} - C_{pStör}F_{pStör}e^{-pu_{K2}} = 0 \qquad (1)$$

$$B_{pStör}e^{pu_{K2}} - E_{Ip}e^{pu_{K2}} - D_{pStör}E_{pStör}e^{pu_{K2}} - D_{pStör}F_{pStör}e^{-pu_{K2}} = 0 \qquad (2)$$

First condition

for $u = u_{K1}$

$$\left(\frac{\partial A_2}{\partial v} - \frac{\partial A_3}{\partial v}\right)_{u_{K1}} = 0 = \left[\left(\frac{\partial A_{eIV}}{\partial v} + \frac{\partial A_{IIStör}}{\partial v}\right) - \frac{\partial A_{IIIStör}}{\partial v}\right]_{u_{K1}}$$

$$0 = \left[\left(\frac{\partial}{\partial v}[D_{IVp}\cos pv + E_{IVp}\sin pv]e^{-pu} + \frac{\partial}{\partial v}\left([C_{pStör}\cos pv + D_{pStör}\sin pv][E_{pStör}e^{pu} + F_{pStör}e^{-pu}]\right)\right) + -\frac{\partial}{\partial v}\left([G_{pStör}\cos pv + H_{pStör}\sin pv]e^{-pu}\right)\right]_{u_{K1}}$$

$$p[-D_{IVp}\sin pv + E_{IVp}\cos pv]e^{-pu} + p[-C_{pStör}\sin pv + D_{pStör}\cos pv][E_{pStör}e^{pu} + F_{pStör}e^{-pu}] \ldots$$
$$-p[-G_{pStör}\sin pv + H_{pStör}\cos pv]e^{-pu} = 0$$

For $u = u_{K1}$ and sorting to sinus and cosines, dividing by p result in

$$\sin pv\ \{-D_{IVp}e^{-pu_{K1}} - C_{pStör}[E_{pStör}e^{pu_{K1}} + F_{pStör}e^{-pu_{K1}}] + G_{pStör}e^{-pu_{K1}}\} = 0$$

$$\cos pv\ \{E_{IVp}e^{-pu_{K1}} + D_{pStör}[E_{pStör}e^{pu_{K1}} + F_{pStör}e^{-pu_{K1}}] - H_{pStör}e^{-pu_{K1}}\} = 0$$

We have now the second two equations for the coefficient.

$$D_{IVp}e^{-pu_{K1}} + C_{pStör}E_{pStör}e^{pu_{K1}} + C_{pStör}F_{pStör}e^{-pu_{K1}} - G_{pStör}e^{-pu_{K1}} = 0 \qquad (3)$$

$$E_{IVp}e^{-pu_{K1}} + D_{pStör}E_{pStör}e^{pu_{K1}} + D_{pStör}F_{pStör}e^{-pu_{K1}} - H_{pStör}e^{-pu_{K1}} = 0 \qquad (4)$$

Second condition for tangent component

for $u = u_{K2}$

$$\left(\frac{1}{\mu_2}\frac{\partial A_1}{\partial u} - \frac{1}{\mu_0}\frac{\partial A_2}{\partial u}\right)_{u_{K2}} = 0 = \left[\frac{1}{\mu_2}\frac{\partial A_{IStör}}{\partial u} - \frac{1}{\mu_0}\left(\frac{\partial A_{eI}}{\partial u} + \frac{\partial A_{IIStör}}{\partial u}\right)\right]_{u_{K2}}$$

$$0 = \left[\frac{1}{\mu_2}\frac{\partial}{\partial u}\left([A_{pStör}\cos pv + B_{pStör}\sin pv]e^{pu}\right) - \frac{1}{\mu_0}\left(\frac{\partial}{\partial u}\{[D_{Ip}\cos pv + E_{Ip}\sin pv]e^{pu}\} + \frac{\partial}{\partial u}\{[C_{pStör}\cos pv + D_{pStör}\sin pv][E_{pStör}e^{pu} + F_{pStör}e^{-pu}]\}\right)\right]_{u_{K2}}$$

$$-\frac{1}{\mu_0}\{p[D_{Ip}\cos pv + E_{Ip}\sin pv]e^{pu} + p[C_{pStör}\cos pv + D_{pStör}\sin pv][E_{pStör}e^{pu} - F_{pStör}e^{-pu}]\} + \frac{1}{\mu_2}p[A_{pStör}\cos pv + B_{pStör}\sin pv]e^{pu} = 0 \quad \text{for } u = u_{K2}$$

For $u = u_{K2}$ and sorting to sinus/ cosines, dividing by p, multiplying by μ_0 will result in

$$\cos pv\left\{D_{Ip}\,e^{pu_{K2}} + C_{pStör}\left[E_{pStör}e^{pu_{K2}} - F_{pStör}e^{-pu_{K2}}\right] - \frac{\mu_0}{\mu_2}A_{pStör}e^{pu_{K2}}\right\} = 0$$

$$\sin pv\left\{E_{Ip}\,e^{pu_{K2}} + D_{pStör}\left[E_{pStör}e^{pu_{K2}} - F_{pStör}e^{-pu_{K2}}\right] - \frac{\mu_0}{\mu_2}B_{pStör}e^{pu_{K2}}\right\} = 0$$

We have now the third dual set of equations for the coefficient.

$$D_{Ip}\,e^{pu_{K2}} + C_{pStör}\,E_{pStör}e^{pu_{K2}} - C_{pStör}F_{pStör}e^{-pu_{K2}} - \frac{\mu_0}{\mu_2}A_{pStör}e^{pu_{K2}} = 0 \qquad (5)$$

$$E_{Ip}\,e^{pu_{K2}} + D_{pStör}\,E_{pStör}e^{pu_{K2}} - D_{pStör}F_{pStör}e^{-pu_{K2}} - \frac{\mu_0}{\mu_2}B_{pStör}e^{pu_{K2}} = 0 \qquad (6)$$

Second condition

for $u = u_{K1}$

$$\left(\frac{1}{\mu_0}\frac{\partial A_2}{\partial u} - \frac{1}{\mu_1}\frac{\partial A_3}{\partial u}\right)_{u_{K1}} = 0 = \left[\frac{1}{\mu_0}\left(\frac{\partial A_{eIV}}{\partial u} + \frac{\partial A_{IIStör}}{\partial u}\right) - \frac{1}{\mu_1}\frac{\partial A_{IIIStör}}{\partial u}\right]_{u_{K1}}$$

$$0 = \left[\frac{1}{\mu_0}\left(\frac{\partial}{\partial u}\{[D_{IVp}\cos pv + E_{IVp}\sin pv]e^{-pu}\} + \frac{\partial}{\partial u}\{[C_{pStör}\cos pv + D_{pStör}\sin pv][E_{pStör}e^{pu} + F_{pStör}e^{-pu}]\}\right) - \frac{1}{\mu_1}\frac{\partial}{\partial u}\{[G_{pStör}\cos pv + H_{pStör}\sin pv]e^{-pu}\}\right]_{u_{K1}}$$

$$\left[\frac{1}{\mu_0}\{-p[D_{IVp}\cos pv + E_{IVp}\sin pv]e^{-pu} + p[C_{pStör}\cos pv + D_{pStör}\sin pv][E_{pStör}e^{pu} - F_{pStör}e^{-pu}]\} + \frac{-1}{\mu_1}(-p)[G_{pStör}\cos pv + H_{pStör}\sin pv]e^{-pu}\right]_{u_{K1}} = 0$$

For $u = u_{K1}$ and sorting to sinus/ cosines, dividing by p, multiplying by μ_0 will result in

$$\cos pv\left\{-D_{IVp}e^{-pu_{K1}} + C_{pStör}\left[E_{pStör}e^{pu_{K1}} - F_{pStör}e^{-pu_{K1}}\right] + \frac{\mu_0}{\mu_1}G_{pStör}\,e^{-pu_{K1}}\right\} = 0$$

$$\sin pv\left\{-E_{IVp}e^{-pu_{K1}} + D_{pStör}\left[E_{pStör}e^{pu_{K1}} - F_{pStör}e^{-pu_{K1}}\right] + \frac{\mu_0}{\mu_1}H_{pStör}\,e^{-pu_{K1}}\right\} = 0$$

We have now the fourth dual equations for the coefficient.

$$-D_{IVp}e^{-pu_{K1}} + C_{pStör}\,E_{pStör}e^{pu_{K1}} - C_{pStör}F_{pStör}e^{-pu_{K1}} + \frac{\mu_0}{\mu_1}G_{pStör}\,e^{-pu_{K1}} = 0 \qquad (7)$$

$$-E_{IVp}e^{-pu_{K1}} + D_{pStör}\,E_{pStör}e^{pu_{K1}} - D_{pStör}F_{pStör}e^{-pu_{K1}} + \frac{\mu_0}{\mu_1}H_{pStör}\,e^{-pu_{K1}} = 0 \qquad (8)$$

By careful viewing you will see that equation 1, 3, 5 an7 as well as 2, 4, 6 and 8 creating a separate independent system of equations.

The following steps will solve this system of equations.

out of 1 & 5 we will get $f_{15} = f_{15}(\, C_{pStör}\, E_{pStör},\, C_{pStör}\, F_{pStör})$

out of 3 & 7 $f_{37} = f_{37}(\, C_{pStör}\, E_{pStör},\, C_{pStör}\, F_{pStör})$

which could be solved to $C_{pStör}\, F_{pStör}$

from (1) $A_{pStör}e^{pu_{K2}} = D_{Ip}e^{pu_{K2}} + C_{pStör}E_{pStör}e^{pu_{K2}} + C_{pStör}F_{pStör}e^{-pu_{K2}}$

from (5) $A_{pStör}e^{pu_{K2}} = \frac{\mu_2}{\mu_0}\left(D_{Ip}\, e^{pu_{K2}} + C_{pStör}\, E_{pStör}e^{pu_{K2}} - C_{pStör}F_{pStör}e^{-pu_{K2}}\right)$

(1) – (5) $0 = \left(\frac{\mu_0-\mu_2}{\mu_0}\right) D_{Ip}e^{pu_{K2}} + \left(\frac{\mu_0-\mu_2}{\mu_0}\right) C_{pStör}E_{pStör}e^{pu_{K2}} + \left(\frac{\mu_0+\mu_2}{\mu_0}\right) C_{pStör}F_{pStör}e^{-pu_{K2}}$

$0 = D_{Ip}e^{pu_{K2}} + C_{pStör}E_{pStör}e^{pu_{K2}} - \left(\frac{\mu_2+\mu_0}{\mu_2-\mu_0}\right) C_{pStör}F_{pStör}e^{-pu_{K2}}$ multiplying $e^{pu_{K1}}$

$$C_{pStör}E_{pStör}e^{pu_{K2}}e^{pu_{K1}} = -D_{Ip}e^{pu_{K2}}e^{pu_{K1}} + \left(\frac{\mu_2+\mu_0}{\mu_2-\mu_0}\right) C_{pStör}F_{pStör}e^{-pu_{K2}}e^{pu_{K1}} \quad \text{(a)}$$

Similar with equation 3 and 7

from (3) $G_{pStör}e^{-pu_{K1}} = D_{IVp}e^{-pu_{K1}} + C_{pStör}E_{pStör}e^{pu_{K1}} + C_{pStör}F_{pStör}e^{-pu_{K1}}$

from (7) $G_{pStör}\, e^{-pu_{K1}} = \frac{\mu_1}{\mu_0}\left(D_{IVp}e^{-pu_{K1}} - C_{pStör}\, E_{pStör}e^{pu_{K1}} + C_{pStör}F_{pStör}e^{-pu_{K1}}\right)$

(3) – (7) $0 = \left(\frac{\mu_0-\mu_1}{\mu_0}\right) D_{IVp}e^{-pu_{K1}} + \left(\frac{\mu_0+\mu_1}{\mu_0}\right) C_{pStör}\, E_{pStör}e^{pu_{K1}} + \left(\frac{\mu_0-\mu_1}{\mu_0}\right) C_{pStör}F_{pStör}e^{-pu_{K1}}$

$0 = \left(\frac{\mu_0-\mu_1}{\mu_0+\mu_1}\right) D_{IVp}e^{-pu_{K1}} + C_{pStör}\, E_{pStör}e^{pu_{K1}} + \left(\frac{\mu_0-\mu_1}{\mu_0+\mu_1}\right) C_{pStör}F_{pStör}e^{-pu_{K1}}$ $e^{pu_{K2}}$

$$C_{pStör}\, E_{pStör}e^{pu_{K1}}e^{pu_{K2}} = \left(\frac{\mu_1-\mu_0}{\mu_1+\mu_0}\right) D_{IVp}e^{-pu_{K1}}e^{pu_{K2}} + \left(\frac{\mu_1-\mu_0}{\mu_1+\mu_0}\right) C_{pStör}F_{pStör}e^{-pu_{K1}}e^{pu_{K2}} \quad \text{(b)}$$

Set (a) = (b)

$$-D_{Ip}e^{pu_{K2}}e^{pu_{K1}} + \left(\frac{\mu_2+\mu_0}{\mu_2-\mu_0}\right) C_{pStör}F_{pStör}e^{-pu_{K2}}e^{pu_{K1}} = \left(\frac{\mu_1-\mu_0}{\mu_1+\mu_0}\right) D_{IVp}e^{-pu_{K1}}e^{pu_{K2}} + \left(\frac{\mu_1-\mu_0}{\mu_1+\mu_0}\right) C_{pStör}F_{pStör}e^{-pu_{K1}}e^{pu_{K2}}$$

Keep C...F... on the left side resulted in

$$C_{pStör}F_{pStör}\left\{e^{p(u_{K1}-u_{K2})}\left(\frac{\mu_2+\mu_0}{\mu_2-\mu_0}\right) - e^{-p(u_{K1}-u_{K2})}\left(\frac{\mu_1-\mu_0}{\mu_1+\mu_0}\right)\right\} = D_{Ip}e^{p(u_{K1}+u_{K2})} + \left(\frac{\mu_1-\mu_0}{\mu_1+\mu_0}\right) D_{IVp}e^{-p(u_{K1}-u_{K2})}$$

Therefore the first coefficient:

$$C_{pStör}F_{pStör} = \frac{D_{Ip}e^{p(u_{K1}+u_{K2})} + \left(\frac{\mu_1-\mu_0}{\mu_1+\mu_0}\right) D_{IVp}e^{-p(u_{K1}-u_{K2})}}{\left(\frac{\mu_2+\mu_0}{\mu_2-\mu_0}\right) e^{p(u_{K1}-u_{K2})} - \left(\frac{\mu_1-\mu_0}{\mu_1+\mu_0}\right) e^{-p(u_{K1}-u_{K2})}} \quad \text{(II.4.6)}$$

The other coefficient can be calculated as follows:

$$C_{pStör}E_{pStör} = \left(D_{IVp} + C_{pStör}F_{pStör}\right)\left(\frac{\mu_1 - \mu_0}{\mu_1 + \mu_0}\right) e^{-2pu_{K1}} \quad \text{out of (b)} \quad (II.4.7)$$

$$A_{pStör} = D_{Ip} + C_{pStör}E_{pStör} + C_{pStör}F_{pStör}e^{-2pu_{K2}} \quad \text{from (1)} \quad (II.4.8)$$

$$G_{pStör} = D_{IVp} + C_{pStör}E_{pStör}e^{2pu_{K1}} + C_{pStör}F_{pStör} \quad \text{and with (3)} \quad (II.4.9)$$

Due to the fact that both systems of equations look rather equal, we get the other coefficient for the other equations by comparison

$A_{pStör} \rightarrow B_{pStör}$ $\quad D_{Ip} \rightarrow E_{Ip}$ $\quad C_{pStör}E_{pStör} \rightarrow D_{pStör}E_{pStör}$

$G_{pStör} \rightarrow H_{pStör}$ $\quad D_{IVp} \rightarrow E_{IVp}$ $\quad C_{pStör}F_{pStör} \rightarrow D_{pStör}F_{pStör}$

$$D_{pStör}F_{pStör} = \frac{E_{Ip}e^{p(u_{K1}+u_{K2})} + \left(\frac{\mu_1 - \mu_0}{\mu_1 + \mu_0}\right) E_{IVp}e^{-p(u_{K1}-u_{K2})}}{\left(\frac{\mu_2 + \mu_0}{\mu_2 - \mu_0}\right) e^{p(u_{K1}-u_{K2})} - \left(\frac{\mu_1 - \mu_0}{\mu_1 + \mu_0}\right) e^{-p(u_{K1}-u_{K2})}} \quad (II.4.10)$$

$$D_{pStör}E_{pStör} = \left(E_{IVp} + D_{pStör}F_{pStör}\right)\left(\frac{\mu_1 - \mu_0}{\mu_1 + \mu_0}\right) e^{-2pu_{K1}} \quad (II.4.11)$$

$$B_{pStör} = E_{Ip} + D_{pStör}E_{pStör} + D_{pStör}F_{pStör}e^{-2pu_{K2}} \quad (II.4.12)$$

$$H_{pStör} = E_{IVp} + D_{pStör}E_{pStör}e^{2pu_{K1}} + D_{pStör}F_{pStör} \quad (II.4.13)$$

With these coefficient could be the “Stör” potential/ field calculated based on equation (II:4.1) for volume I

$$A_{IStör} = \sum_{p=1}^{\infty}\left[A_{pStör}\cos pv + B_{pStör}\sin pv\right] e^{pu} \quad (II.4.1)$$

The field ansatz for volume II will be by multiplication of trigonometric with e-function as follow

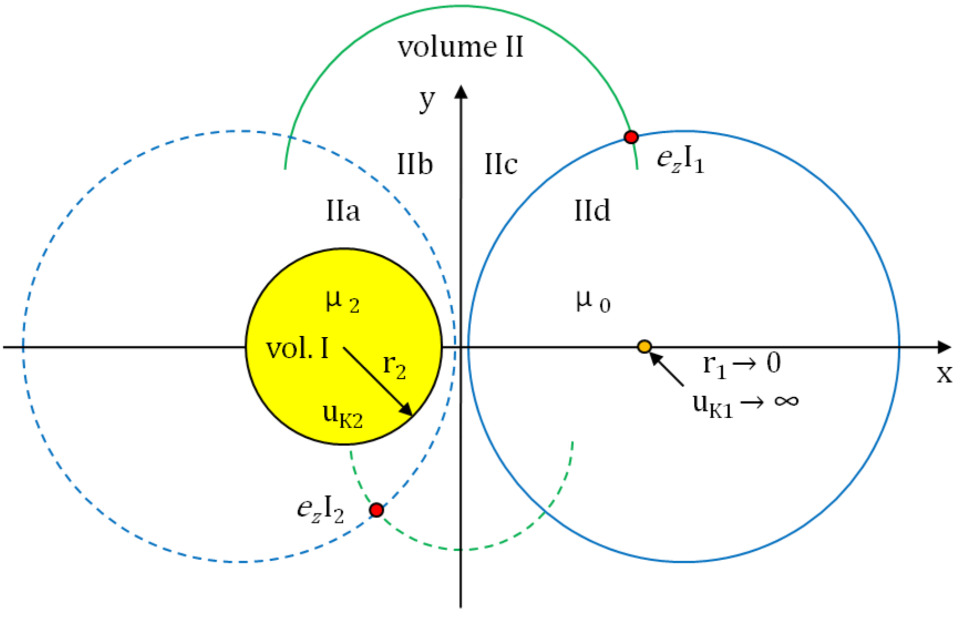
$$A_{IIStör} = \left\{\begin{matrix} A_{eI} \\ A_{eII} \\ A_{eIII} \\ A_{eIV} \end{matrix}\right\} + \sum_{p=1}^{\infty}\left[\begin{matrix} \left(C_{pStör}\,E_{pStör}\cos pv + D_{pStör}\,E_{pStör}\sin pv\right)e^{pu} + \\ +\left(C_{pStör}\,F_{pStör}\cos pv + D_{pStör}\,F_{pStör}\sin pv\right)e^{-pu} \end{matrix}\right] \quad (II.4.2)$$

and (II.4.3) for volume III

$$A_{IIIStör} = \sum_{p=1}^{\infty}\left[G_{pStör}\cos pv + H_{pStör}\sin pv\right] e^{-pu} \quad (II.4.3)$$

II.5 Example: conductor in front of one permeable cylinder

One of these permeable cylinder shall be disappearing with $r_1 = 0$ and equation (I.2.2) we get $u_{K1} = \sinh^{-1}\left(\frac{a}{r_1}\right) \to \infty$



The different coefficient will be based on II.4.6 … 13

$$\lim_{u_{K1}\to\infty} C_{pSt\ddot{o}r} F_{pSt\ddot{o}r} = \left(\frac{\mu_2 - \mu_0}{\mu_2 + \mu_0}\right) D_{Ip}\, e^{2pu_{K2}}$$

$$\lim_{u_{K1}\to\infty} C_{pSt\ddot{o}r} E_{pSt\ddot{o}r} = 0$$

$$\lim_{u_{K1}\to\infty} A_{pSt\ddot{o}r} = D_{Ip} + \left(\frac{\mu_2 - \mu_0}{\mu_2 + \mu_0}\right) D_{Ip}\, e^{2pu_{K2}} e^{-2pu_{K2}} = D_{Ip} \frac{2\mu_2}{\mu_2 + \mu_0}$$

$$\lim_{u_{K1}\to\infty} G_{pSt\ddot{o}r} = D_{IVp} + C_{pSt\ddot{o}r} F_{pSt\ddot{o}r} = D_{IVp} + D_{Ip} \left(\frac{\mu_2 - \mu_0}{\mu_2 + \mu_0}\right) e^{2pu_{K2}}$$

Similar we will get the other coefficients by comparison

$$D_{pSt\ddot{o}r} F_{pSt\ddot{o}r} = \left(\frac{\mu_2 - \mu_0}{\mu_2 + \mu_0}\right) E_{Ip}\, e^{2pu_{K2}}$$

$$D_{pSt\ddot{o}r} E_{pSt\ddot{o}r} = 0$$

$$B_{pSt\ddot{o}r} = E_{Ip} \frac{2\mu_2}{\mu_2 + \mu_0}$$

$$H_{pSt\ddot{o}r} = E_{IVp} + E_{Ip} \left(\frac{\mu_2 - \mu_0}{\mu_2 + \mu_0}\right) e^{2pu_{K2}}$$

Therefore the “Stör” potentials in both volumes

$$A_{ISt\ddot{o}r} = \frac{2\mu_2}{\mu_2 + \mu_0} \sum_{p=1}^{\infty} \left[D_{Ip} \cos pv + E_{Ip} \sin pv\right] e^{pu} \qquad \text{(II.5.1)}$$

$$A_{IISt\ddot{o}r} = A_e + \left(\frac{\mu_2 - \mu_0}{\mu_2 + \mu_0}\right) \sum_{p=1}^{\infty} \left[e^{2pu_{K2}} \left(D_{Ip} \cos pv + E_{Ip} \sin pv\right) e^{-pu}\right] \qquad \text{(II.5.2)}$$

Note: Don’t forget the exciting potential A_e in volume II with the coefficient of section II.3

For the special case of just one live conductor with current I in front of a permeable cylinder we will get the following situation.

Conductor: $x_1 = y_1 = 0$

$u_1 = 0$

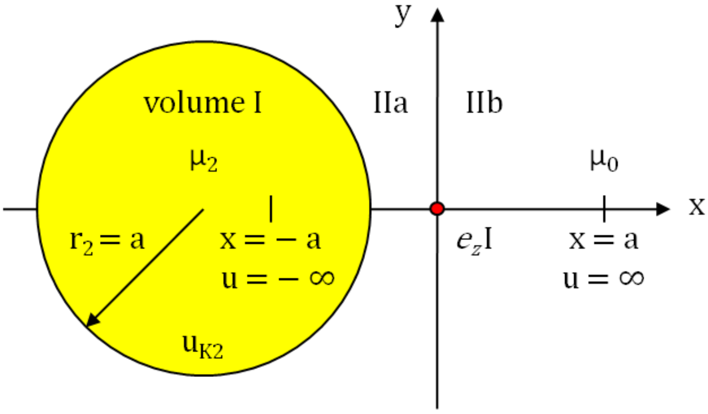
$v_1 = \pi$

Cylinder: $r_2 = a$

$u_{2K} = -\sinh^{-1}(a/r_2)$

$u_{K2} = -\sinh^{-1} 1$

$u_{K2} = -0{,}88137$



The potential in all three partial volumes is as follows:

Volume I $-\infty < u < u_{K2}$ based on (II.5.1)

$$A_{IStör} = \frac{2\mu_2}{\mu_2+\mu_0} \sum_{p=1}^{\infty}\left[D_{Ip}\cos pv + E_{Ip}\sin pv\right] e^{pu}$$

Volume IIa $u_{K2} < u < 0$ based on (II.5.2) and (II.1.6b)

$$A_{IIaStör} = A_{eIIa} + \left(\frac{\mu_2-\mu_0}{\mu_2+\mu_0}\right) \sum_{p=1}^{\infty}\left[e^{2pu_{K2}} \left(D_{Ip}\cos pv + E_{Ip}\sin pv\right)e^{-pu}\right]$$

$$A_{eIIa} = \sum_{p=1}^{\infty}\left[\left(A_p + B_p\right)\cos pv + C_p \sin pv\right] e^{pu}$$

Volume IIb $0 < u < \infty$ based on (II.5.2) and (II.1.6c)

$$A_{IIbStör} = A_{eIIb} + \left(\frac{\mu_2-\mu_0}{\mu_2+\mu_0}\right) \sum_{p=1}^{\infty}\left[e^{2pu_{K2}} \left(D_{Ip}\cos pv + E_{Ip}\sin pv\right)e^{-pu}\right]$$

$$A_{eIIb} = \sum_{p=1}^{\infty}\left[\left(A_p + B_p e^{2pu_1}\right)\cos pv + C_p e^{2pu_1} \sin pv\right]e^{-pu}$$

The coefficients are with (II.3.5/6a)

$$D_{Ip} = A_p + B_p \qquad E_{Ip} = C_p$$

And finally with the exciting basic coefficients based on (II.1.5a/b/c)

$$A_p = -\frac{\mu_0 I_1}{2\pi}\frac{1}{p} \qquad B_p = \frac{\mu_0 I_1}{2\pi}\frac{\cos pv_1}{p e^{pu_1}} \qquad C_p = \frac{\mu_0 I_1}{2\pi}\frac{\sin pv_1}{p e^{pu_1}}$$

For the conductor coordinate $x_1 = y_1 = 0$ and therefore $v_1 = \pi$ the coefficient C_p will be always zero as well as E_{Ip}. The cosines function will be either + or – 1 equivalent to $(-1)^p$ and with $u_1 = 0$ the exponential function will be always one, therefore

$$D_{Ip} = A_p + B_p = -\frac{\mu_0 I_1}{2\pi}\frac{1}{p} + \frac{\mu_0 I_1}{2\pi}\frac{(-1)^p}{p} = -\frac{\mu_0 I_1}{2\pi}\frac{1}{p}\left[1-(-1)^p\right]$$

Therefore the magnetic potential in the three volumes:

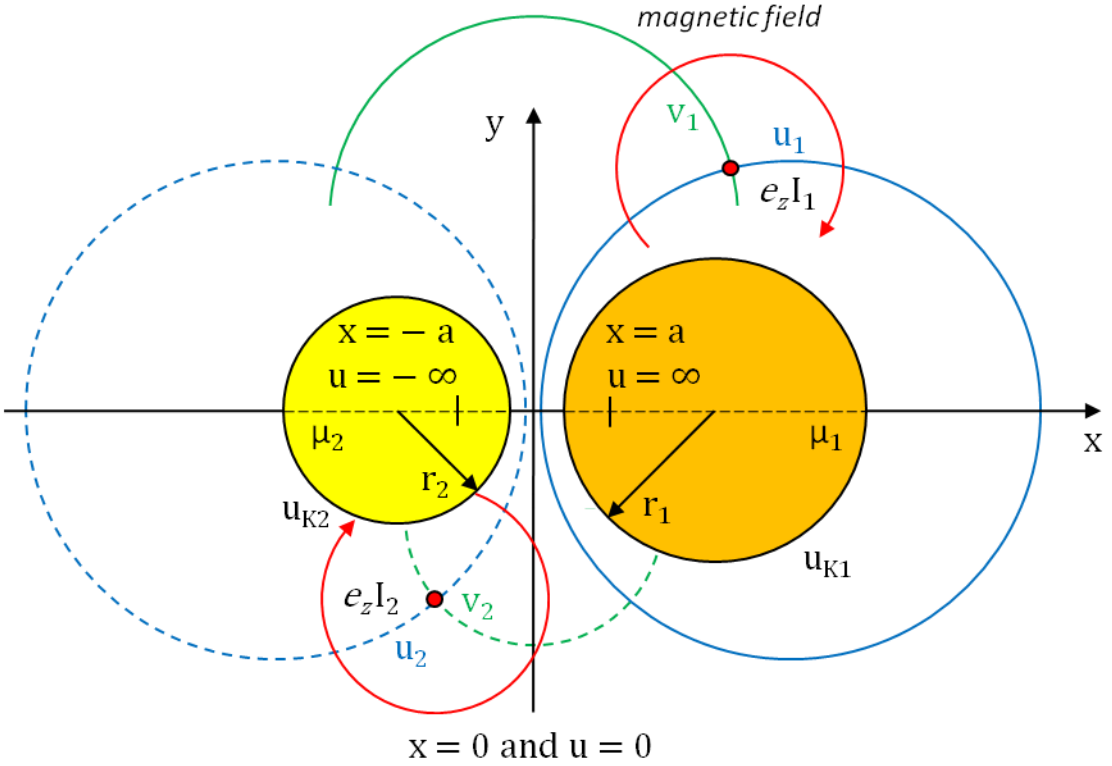
$$A_{IStör} = \frac{2\mu_2}{\mu_2+\mu_0} \sum_{p=1}^{\infty}\left[D_{Ip}\cos pv\right] e^{pu} \qquad \text{(II.5.3)}$$

$$A_{IIaStör} = \sum_{p=1}^{\infty}\left[D_{Ip}\cos pv\right] e^{pu} + \left(\frac{\mu_2-\mu_0}{\mu_2+\mu_0}\right) \sum_{p=1}^{\infty}\left[e^{2pu_{K2}} \left(D_{Ip}\cos pv\right)e^{-pu}\right] \qquad \text{(II.5.4)}$$

$$A_{IIbStör} = \sum_{p=1}^{\infty}\left[D_{Ip}\cos pv\right]e^{-pu} + \left(\frac{\mu_2-\mu_0}{\mu_2+\mu_0}\right) \sum_{p=1}^{\infty}\left[e^{2pu_{K2}} \left(D_{Ip}\cos pv\right)e^{-pu}\right] \qquad \text{(II.5.5)}$$

III. Force on the conductors

III.1 Force on two live conductors



The force $\boldsymbol{K}$ of a conductor with current density $\boldsymbol{G}$ within an outer field of induction $\boldsymbol{B}$ will be calculated as volume integral

$$\vec{K} = \iiint_v (\vec{G} \times \vec{B}) dv$$

For the linear case of a thin conductor with Current I in z-direction the volume integral will be replaced by a contour integration

$$\vec{K} = I \oint_c (\overrightarrow{dr} \times \vec{B})$$

For a plane problem with current in z-direction the differential length will be $dr = e_z$ dz and the force on the conductor per length unit will be

$$\frac{\overrightarrow{dK}}{dz} = I\,(\overrightarrow{e_z} \times \vec{B})$$

Here is the induction B at the location of the conductor induced by the other conductor and the “Stör” field. The induction will be based on the rotation of magnetic potential A.

For a z-directed vector potential $\vec{A} = \overrightarrow{e_z}\,A(u, v)$ is the induction the result of

$$\vec{B} = rot\,\vec{A} = \frac{1}{h}\left[\overrightarrow{e_u}\,\frac{\partial A}{\partial v} - \overrightarrow{e_v}\,\frac{\partial A}{\partial u}\right]$$

The exciting vector field $\boldsymbol{A}$ of two conductors is described in para II.1.6/ II.2.2 and the “Stör” potential is defined in section II.4.

Conductor 1:

$-\infty < u < 0$

$$\overrightarrow{B_{eI}} = \frac{1}{h}\begin{Bmatrix} \overrightarrow{e_u}\,\frac{\partial}{\partial v}\left[\sum_{p=1}^{\infty}\left[(A_p + B_p)\cos pv + C_p \sin pv\right]e^{pu}\right] - \ldots \\ -\overrightarrow{e_v}\,\frac{\partial}{\partial u}\sum_{p=1}^{\infty}\left[(A_p + B_p)\cos pv + C_p \sin pv\right]e^{pu} \end{Bmatrix}$$

$$\overrightarrow{B_{eI}} = \frac{1}{h}\begin{Bmatrix} \overrightarrow{e_u}\left[\sum_{p=1}^{\infty} p\left[-(A_p + B_p)\sin pv + C_p \cos pv\right]e^{pu}\right] - \ldots \\ -\overrightarrow{e_v}\sum_{p=1}^{\infty} p\left[(A_p + B_p)\cos pv + C_p \sin pv\right]e^{pu} \end{Bmatrix} \qquad \text{(a)}$$

$0 < u < u_1$

$$\overrightarrow{B_{eII}} = \frac{1}{h}\begin{Bmatrix} \overrightarrow{e_u}\,\frac{\partial}{\partial v}\sum_{p=1}^{\infty}\left[(B_p\, e^{pu} + A_p e^{-pu})\cos pv + C_p\, e^{pu}\sin pv\right] - \ldots \\ -\overrightarrow{e_v}\,\frac{\partial}{\partial u}\sum_{p=1}^{\infty}\left[(B_p\, e^{pu} + A_p e^{-pu})\cos pv + C_p\, e^{pu}\sin pv\right] \end{Bmatrix}$$

$$\overrightarrow{B_{eII}} = \frac{1}{h}\begin{Bmatrix} \overrightarrow{e_u}\sum_{p=1}^{\infty} p\left[-(B_p\, e^{pu} + A_p e^{-pu})\sin pv + C_p\, e^{pu}\cos pv\right] - \ldots \\ -\overrightarrow{e_v}\sum_{p=1}^{\infty} p\left[(B_p\, e^{pu} - A_p e^{-pu})\cos pv + C_p\, e^{pu}\sin pv\right] \end{Bmatrix} \qquad \text{(b)}$$

$u_1 < u < \infty$

$$\overrightarrow{B_{eIII}} = \frac{1}{h}\begin{Bmatrix} \overrightarrow{e_u}\,\frac{\partial}{\partial v}\sum_{p=1}^{\infty}\left[(A_p + B_p e^{2pu_1})\cos pv + C_p e^{2pu_1}\sin pv\right]e^{-pu} - \ldots \\ -\overrightarrow{e_v}\,\frac{\partial}{\partial u}\sum_{p=1}^{\infty}\left[(A_p + B_p e^{2pu_1})\cos pv + C_p e^{2pu_1}\sin pv\right]e^{-pu} \end{Bmatrix}$$

$$\overrightarrow{B_{eIII}} = \frac{1}{h}\begin{Bmatrix} \overrightarrow{e_u}\sum_{p=1}^{\infty} p\left[-(A_p + B_p e^{2pu_1})\sin pv + C_p e^{2pu_1}\cos pv\right]e^{-pu} - \ldots \\ -\overrightarrow{e_v}\sum_{p=1}^{\infty}(-1)p\left[(A_p + B_p e^{2pu_1})\cos pv + C_p e^{2pu_1}\sin pv\right]e^{-pu} \end{Bmatrix} \qquad \text{(c)}$$

Conductor 2:

$-\infty < u < u_2$

$$\overrightarrow{B^*_{eIII}} = \frac{1}{h}\begin{Bmatrix} \overrightarrow{e_u}\,\frac{\partial}{\partial v}\sum_{p=1}^{\infty}\left[(A^*_p + B^*_p e^{-2pu_2})\cos pv + C^*_p e^{-2pu_2}\sin pv\right]e^{+pu} - \ldots \\ -\overrightarrow{e_v}\,\frac{\partial}{\partial u}\sum_{p=1}^{\infty}\left[(A^*_p + B^*_p e^{-2pu_2})\cos pv + C^*_p e^{-2pu_2}\sin pv\right]e^{+pu} \end{Bmatrix}$$

$$\overrightarrow{B^*_{eIII}} = \frac{1}{h}\begin{Bmatrix} \overrightarrow{e_u}\sum_{p=1}^{\infty} p\left[-(A^*_p + B^*_p e^{-2pu_2})\sin pv + C^*_p e^{-2pu_2}\cos pv\right]e^{+pu} - \ldots \\ -\overrightarrow{e_v}\sum_{p=1}^{\infty} p\left[(A^*_p + B^*_p e^{-2pu_2})\cos pv + C^*_p e^{-2pu_2}\sin pv\right]e^{+pu} \end{Bmatrix} \qquad \text{(d)}$$

$u_2 < u < 0$

$$\overrightarrow{B^*_{eII}} = \frac{1}{h}\begin{Bmatrix} \overrightarrow{e_u}\,\frac{\partial}{\partial v}\sum_{p=1}^{\infty}\left[(B^*_p\, e^{-pu} + A^*_p e^{+pu})\cos pv + C^*_p\, e^{-pu}\sin pv\right] - \ldots \\ -\overrightarrow{e_v}\,\frac{\partial}{\partial u}\sum_{p=1}^{\infty}\left[(B^*_p\, e^{-pu} + A^*_p e^{+pu})\cos pv + C^*_p\, e^{-pu}\sin pv\right] \end{Bmatrix}$$

$$\overrightarrow{B^*_{eII}} = \frac{1}{h}\begin{Bmatrix} \overrightarrow{e_u}\sum_{p=1}^{\infty} p\left[-(B^*_p\, e^{-pu} + A^*_p e^{+pu})\sin pv + C^*_p\, e^{-pu}\cos pv\right] - \ldots \\ -\overrightarrow{e_v}\sum_{p=1}^{\infty} p\left[(-B^*_p\, e^{-pu} + A^*_p e^{+pu})\cos pv - C^*_p\, e^{-pu}\sin pv\right] \end{Bmatrix} \qquad \text{(e)}$$

$0 < u < \infty$

$$\overrightarrow{B^*_{eI}} = \frac{1}{h}\begin{Bmatrix} \overrightarrow{e_u}\,\frac{\partial}{\partial v}\sum_{p=1}^{\infty}\left[(A^*_p + B^*_p)\cos pv + C^*_p \sin pv\right]e^{-pu} - \ldots \\ -\overrightarrow{e_v}\,\frac{\partial}{\partial u}\sum_{p=1}^{\infty}\left[(A^*_p + B^*_p)\cos pv + C^*_p \sin pv\right]e^{-pu} \end{Bmatrix}$$

$$\overrightarrow{B^*_{eI}} = \frac{1}{h}\begin{Bmatrix} \overrightarrow{e_u}\sum_{p=1}^{\infty} p\left[-(A^*_p + B^*_p)\sin pv + C^*_p \cos pv\right]e^{-pu} - \ldots \\ -\overrightarrow{e_v}\sum_{p=1}^{\infty}(-p)\left[(A^*_p + B^*_p)\cos pv + C^*_p \sin pv\right]e^{-pu} \end{Bmatrix} \qquad \text{(f)}$$

"Stör" induction

$$-\infty < u < u_{K2}$$

$$\overrightarrow{B_{IStör}} = \frac{1}{h}\left\{\begin{matrix} \vec{e_u}\,\frac{\partial}{\partial v}\left[\sum_{p=1}^{\infty}\left[A_{pStör}\cos pv + B_{pStör}\sin pv\right]e^{pu}\right] - \ ... \\ -\,\vec{e_v}\,\frac{\partial}{\partial u}\sum_{p=1}^{\infty}\left[A_{pStör}\cos pv + B_{pStör}\sin pv\right]e^{pu} \end{matrix}\right\}$$

$$\overrightarrow{B_{IStör}} = \frac{1}{h}\left\{\begin{matrix} \vec{e_u}\left[\sum_{p=1}^{\infty} p\left[-A_{pStör}\sin pv + B_{pStör}\cos pv\right]e^{pu}\right] - \ ... \\ -\,\vec{e_v}\sum_{p=1}^{\infty} p\left[A_{pStör}\cos pv + B_{pStör}\sin pv\right]e^{pu} \end{matrix}\right\} \qquad (g)$$

$$u_{K2} < u < u_{K1}$$

$$\overrightarrow{B_{IIStör}} = \frac{1}{h}\left\{\begin{matrix} \vec{e_u}\,\frac{\partial}{\partial v}\sum_{p=1}^{\infty}\left[C_{pStör}\cos pv + D_{pStör}\sin pv\right]\left[E_{pStör}e^{pu} + F_{pStör}e^{-pu}\right] - \ ... \\ -\,\vec{e_v}\,\frac{\partial}{\partial u}\sum_{p=1}^{\infty}\left[C_{pStör}\cos pv + D_{pStör}\sin pv\right]\left[E_{pStör}e^{pu} + F_{pStör}e^{-pu}\right] \end{matrix}\right\}$$

$$\overrightarrow{B_{IIStör}} = \frac{1}{h}\left\{\begin{matrix} \vec{e_u}\left[\sum_{p=1}^{\infty} p\left(\begin{matrix}\left(-C_{pStör}E_{pStör}\sin pv + D_{pStör}E_{pStör}\cos pv\right)e^{pu} + \\ \left(-C_{pStör}F_{pStör}\sin pv + D_{pStör}F_{pStör}\cos pv\right)e^{-pu}\end{matrix}\right)\right] \\ -\,\vec{e_v}\left[\sum_{p=1}^{\infty} p\left(\begin{matrix}\left(C_{pStör}E_{pStör}\cos pv + D_{pStör}E_{pStör}\sin pv\right)e^{pu} \\ -\left(C_{pStör}F_{pStör}\cos pv + D_{pStör}F_{pStör}\sin pv\right)e^{-pu}\end{matrix}\right)\right] \end{matrix}\right\} \qquad (h)$$

$$u_{K1} < u < \infty$$

$$\overrightarrow{B_{IIIStör}} = \frac{1}{h}\left\{\begin{matrix} \vec{e_u}\,\frac{\partial}{\partial v}\left[\sum_{p=1}^{\infty}\left[G_{pStör}\cos pv + H_{pStör}\sin pv\right]e^{-pu}\right] - \ ... \\ -\,\vec{e_v}\,\frac{\partial}{\partial u}\sum_{p=1}^{\infty}\left[G_{pStör}\cos pv + H_{pStör}\sin pv\right]e^{-pu} \end{matrix}\right\}$$

$$\overrightarrow{B_{IIIStör}} = \frac{1}{h}\left\{\begin{matrix} \vec{e_u}\left[\sum_{p=1}^{\infty} p\left[-G_{pStör}\sin pv + H_{pStör}\cos pv\right]e^{-pu}\right] - \ ... \\ -\,\vec{e_v}\sum_{p=1}^{\infty}(-1)p\left[G_{pStör}\cos pv + H_{pStör}\sin pv\right]e^{-pu} \end{matrix}\right\} \qquad (i)$$

The calculation of the force on both conductors is now, possible, all coefficients are known (see section II.2 and II.4).

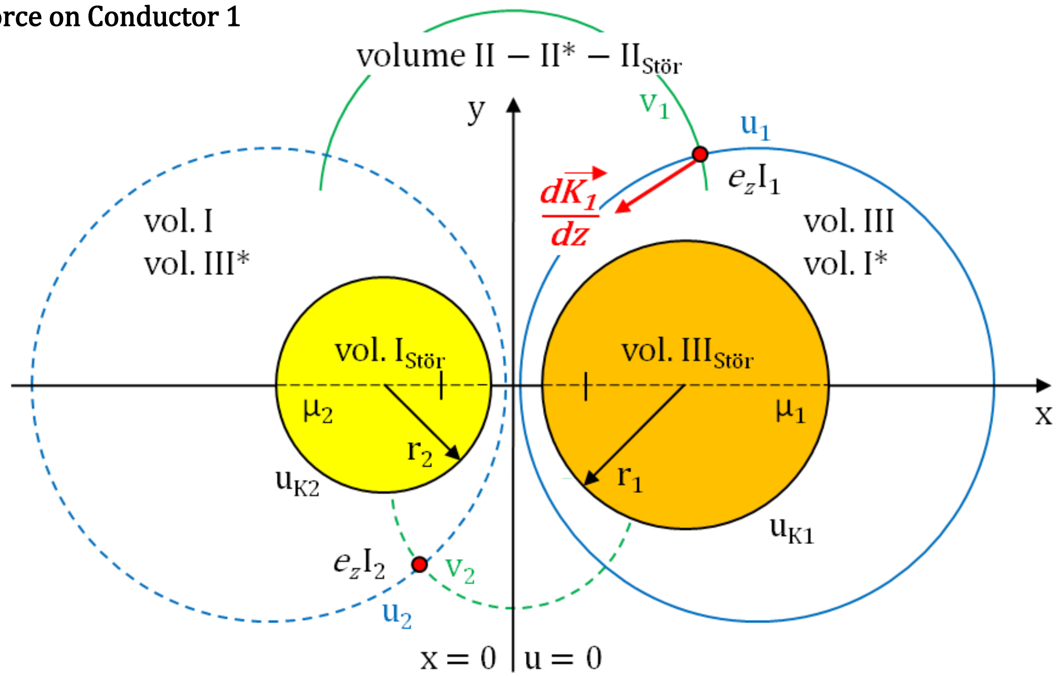
The force per unit length will be in the different locations:

Force on conductor 1 & 2:

$$\frac{\overrightarrow{dK_1}}{dz} = I_1\left(\vec{e_z} \times \vec{B}\right)\Big|_{u_1 v_1} \quad \text{with} \quad \vec{B} = \overrightarrow{B_{eI}^*} + \overrightarrow{B_{IIStör}} \qquad (j)$$

$$\frac{\overrightarrow{dK_2}}{dz} = I_2\left(\vec{e_z} \times \vec{B}\right)\Big|_{u_2 v_2} \quad \text{with} \quad \vec{B} = \overrightarrow{B_{eI}} + \overrightarrow{B_{IIStör}} \qquad (k)$$

Force on Conductor 1



The force on conductor 1 at coordinate u_1, v_1 will be within the field

$$\frac{\overrightarrow{dK_1}}{dz} = I_1 \left(\vec{e_z} \times \vec{B}\right)\Big|_{u_1 v_1} = I_1 \left[\vec{e_z} \times \left(\overrightarrow{B^*_{eI}} + \overrightarrow{B_{II\text{Stör}}}\right)\right]\Big|_{u_1 v_1}$$

with $\overrightarrow{B^*_{eI}} = \vec{e_u}\, B^*_{eIu} - \vec{e_v}\, B^*_{eIv}$ and $\overrightarrow{B_{eII\text{Stör}}} = \vec{e_u}\, B_{eII\text{Stör}u} - \vec{e_v}\, B_{eII\text{Stör}v}$

$$\frac{\overrightarrow{dK_1}}{dz} = I_1 \left\{\vec{e_z} \times \left[\left(\vec{e_u}\, B^*_{eIu} - \vec{e_v}\, B^*_{eIv}\right) + \left(\vec{e_u}\, B_{eII\text{Stör}u} - \vec{e_v}\, B_{eII\text{Stör}v}\right)\right]\right\}\Big|_{u_1 v_1}$$

$$\frac{\overrightarrow{dK_1}}{dz} = I_1 \left\{\vec{e_z} \times \left[\left(\vec{e_u}\left(B^*_{eIu} + B_{eII\text{Stör}u}\right) - \vec{e_v}\left(B^*_{eIv} + B_{eII\text{Stör}v}\right)\right)\right]\right\}\Big|_{u_1 v_1}$$

$$\frac{\overrightarrow{dK_1}}{dz} = I_1 \left[\vec{e_v}\left(B^*_{eIu} + B_{eII\text{Stör}u}\right) + \vec{e_u}\left(B^*_{eIv} + B_{eII\text{Stör}v}\right)\right]\Big|_{u_1 v_1}$$

With equation (f) and (h) the force on conductor 1 will be with h(u,v) for u_1, v_1

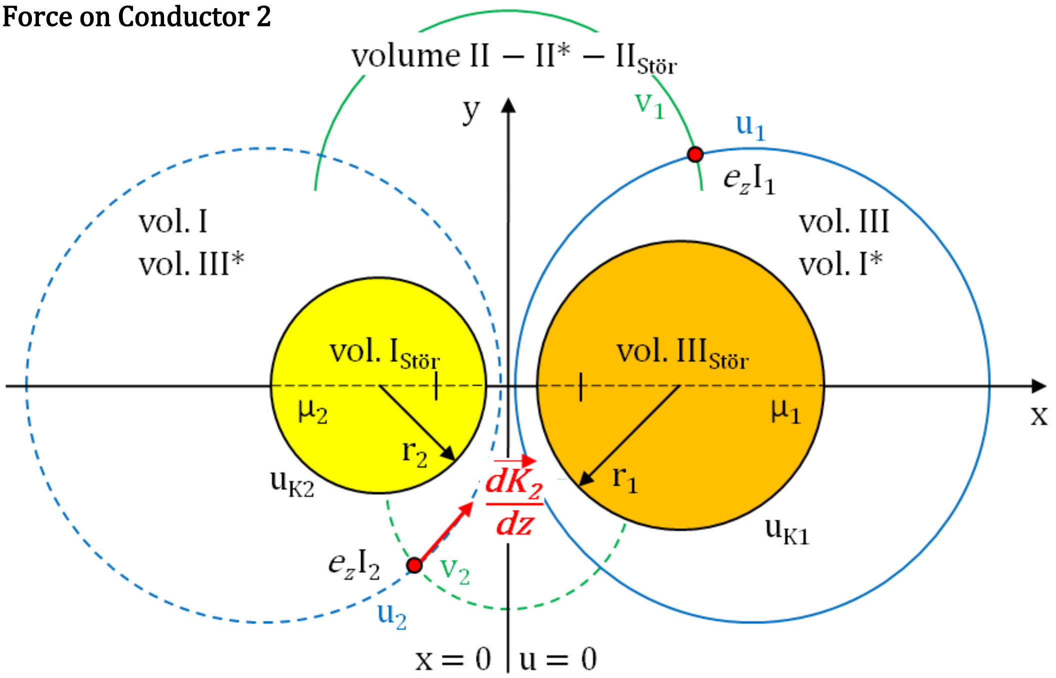
$$\frac{\overrightarrow{dK_1}}{dz} = I_1 \frac{1}{h(u_1,v_1)} \sum_{p=1}^{\infty} p \left\{\begin{array}{l} \vec{e_u}\left[-\left(A^*_p + B^*_p\right)\cos pv - C^*_p \sin pv\right]e^{-pu} \\ + \vec{e_u}\left[\begin{array}{l}\left(C_{p\text{Stör}}\, E_{p\text{Stör}} \cos pv + D_{p\text{Stör}}\, E_{p\text{Stör}} \sin pv\right)e^{pu} \\ -\left(C_{p\text{Stör}}\, F_{p\text{Stör}} \cos pv + D_{p\text{Stör}}\, F_{p\text{Stör}} \sin pv\right)e^{-pu}\end{array}\right] \\ + \vec{e_v}\left[-\left(A^*_p + B^*_p\right)\sin pv + C^*_p \cos pv\right]e^{-pu} \\ + \vec{e_v}\left[\begin{array}{l}\left(-C_{p\text{Stör}}\, E_{p\text{Stör}} \sin pv + D_{p\text{Stör}}\, E_{p\text{Stör}} \cos pv\right)e^{pu} + \\ \left(-C_{p\text{Stör}}\, F_{p\text{Stör}} \sin pv + D_{p\text{Stör}}\, F_{p\text{Stör}} \cos pv\right)e^{-pu}\end{array}\right]\end{array}\right\}$$

$$\boxed{\frac{\overrightarrow{dK_1}}{dz} = \frac{I_1}{h(u_1,v_1)} \sum_{p=1}^{\infty} p \left\{\begin{array}{l} \vec{e_u}\left[-\left(A^*_p + B^*_p + C_{p\text{Stör}}F_{p\text{Stör}}\right)\cos pv_1 - \left(C^*_p + D_{p\text{Stör}}F_{p\text{Stör}}\right)\sin pv_1\right]e^{-pu_1} \\ + \vec{e_u}\left[\left(C_{p\text{Stör}}\, E_{p\text{Stör}} \cos pv_1 + D_{p\text{Stör}}\, E_{p\text{Stör}} \sin pv_1\right)e^{pu_1}\right] \\ + \vec{e_v}\left[-\left(A^*_p + B^*_p + C_{p\text{Stör}}F_{p\text{Stör}}\right)\sin pv_1 + \left(C^*_p + D_{p\text{Stör}}F_{p\text{Stör}}\right)\cos pv_1\right]e^{-pu_1} \\ + \vec{e_v}\left[\left(-C_{p\text{Stör}}\, E_{p\text{Stör}} \sin pv_1 + D_{p\text{Stör}}\, E_{p\text{Stör}} \cos pv_1\right)e^{pu_1}\right]\end{array}\right\}}$$

equation (III.1.1)

The coefficients are defined in section II.2 and II.4

Force on Conductor 2



For conductor two with current I_2 at location u_2, v_2 for the external field $\vec{B} = \overrightarrow{B_{eI}} + \overrightarrow{B_{IIStör}}$

Comparison between induction $\overrightarrow{B_{eI}}$ and $\overrightarrow{B^*_{eI}}$ leads to

$A_p \leftrightarrow A^*_p \qquad B_p \leftrightarrow B^*_p \qquad C_p \leftrightarrow C^*_p$

$\overrightarrow{B_{eIv}} = -\overrightarrow{B^*_{eIv}}$

$e^{pu} \leftrightarrow e^{-pu}$

Under consideration of above mentioned connection for u_2, v_2 we get the force on conductor 2

$$\frac{\overrightarrow{dK_2}}{dz} = \frac{I_2}{h(u_2,v_2)} \sum_{p=1}^{\infty} p \left\{ \begin{array}{c} \overrightarrow{e_u}\left[\left(A_p + B_p + C_{pStör}F_{pStör}\right)\cos pv_2 + \left(C_p + D_{pStör}F_{pStör}\right)\sin pv_2\right]e^{pu_2} \\ -\overrightarrow{e_u}\left[\left(C_{pStör}\,E_{pStör}\cos pv_2 + D_{pStör}\,E_{pStör}\sin pv_2\right)e^{-pu_2}\right] \\ +\overrightarrow{e_v}\left[-\left(A_p + B_p + C_{pStör}F_{pStör}\right)\sin pv_2 + \left(C_p + D_{pStör}F_{pStör}\right)\cos pv_2\right]e^{pu_2} \\ +\overrightarrow{e_v}\left[\left(-C_{pStör}\,E_{pStör}\sin pv_2 + D_{pStör}\,E_{pStör}\cos pv_2\right)e^{-pu_2}\right] \end{array} \right\}$$

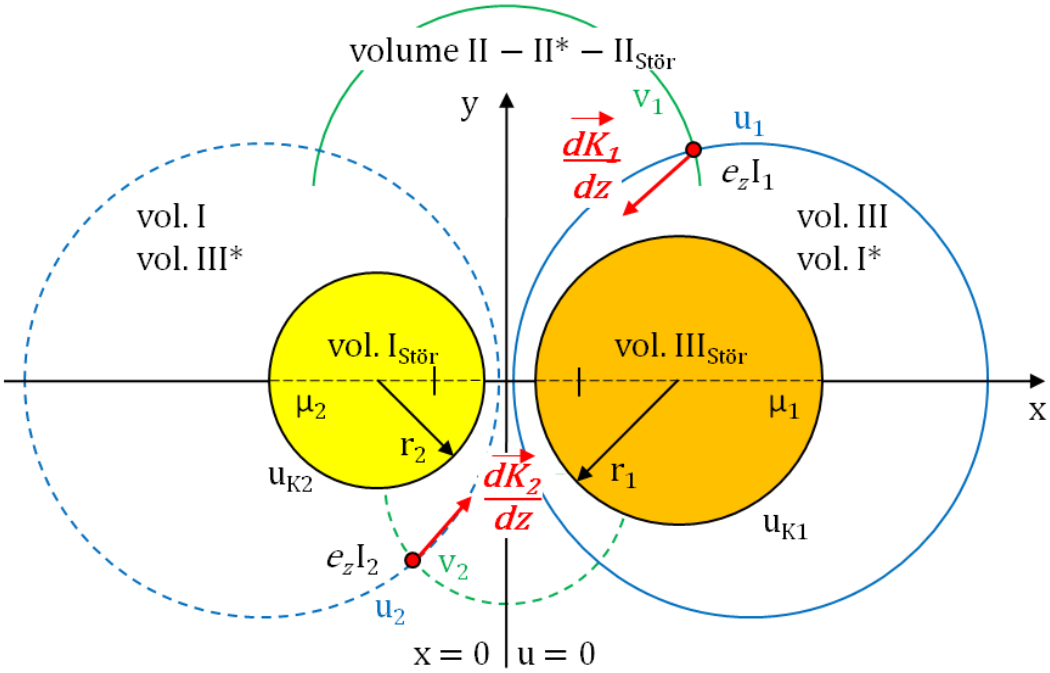
equation (III.1.2)

The coefficients are defined for

A_p, B_p and C_p	(II.1.5a/b/c)	A^*_p, B^*_p and C^*_p	(II.2.1a/b/c)
$C_{pStör}F_{pStör}$	(II.4.6)		
$C_{pStör}E_{pStör}$	(II.4.7)		
$D_{pStör}F_{pStör}$	(II.4.10)		
$D_{pStör}E_{pStör}$	(II.4.11)		

The force on conductor 2 with current I_2 is a result of the other conductor 1 with current I_1 as well as the influence of the "Stör" field by the two permeable cylinders.

Overview of solution:



I_2	Conductor with current	I_1
x_2, y_2 <> u_2, v_2	Location	x_1, y_1 <> u_1, v_1
μ_2 with r_2	permeable cylinder with radius	μ_1 with r_1
A^*_p, B^*_p and C^*_p	exciting field coefficients	A_p, B_p and C_p
	"Stör" field coefficient	
	$C_{pStör}F_{pStör} - C_{pStör}E_{pStör} - D_{pStör}F_{pStör} - D_{pStör}E_{pStör}$	
	$D_{Ip} - E_{Ip} - D_{IVp} - E_{IVp}$	
dK_2/dz	Force on conductor	*dK_1/dz*

Conductor on x-axis:

$y = 0$ $\quad v = \arctan[-2ay/(x^2+y^2-a^2)] = 0$ for $x>a$

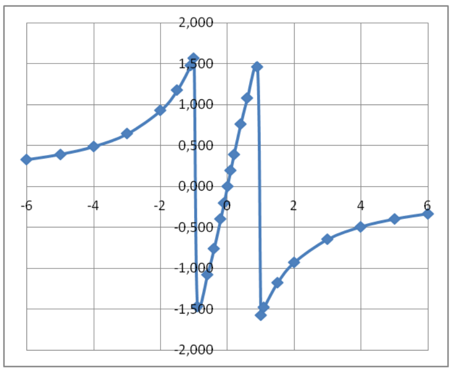
π for $x<a$

$u = \operatorname{arctanh}[2ax/(x^2+a^2)]$

Conductor on y-axis:

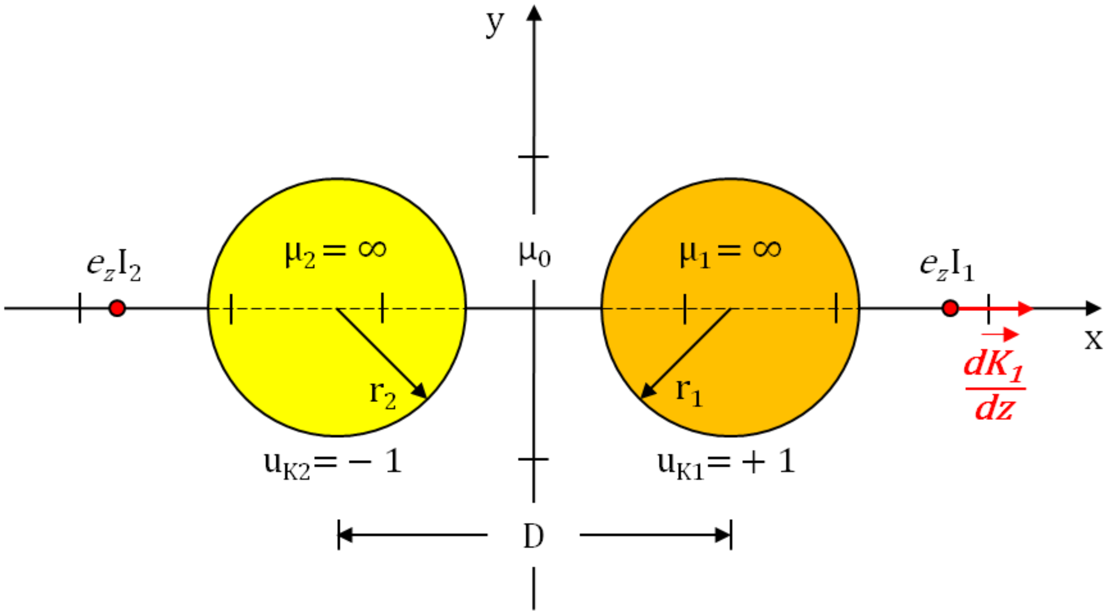
$x = 0$ $\quad u = \operatorname{arctanh}[2ax/(x^2+y^2+a^2)] = 0$

$v = \arctan[-2ay/(y^2-a^2)]$



III.2 Example: Force on two conductors on x-axis

We would like to analyse the following situation.



Two conductors with current I_1, I_2 are located at the x-axis, the cylinders are high permeable with radius r in a distance D:

D = 2,626 (metre, yard, …) r = 0,8509 (metre, yard, …)

The geometric constant a of the bipolar-cylindrical coordinates will be (see I.2.1)

$$a = \sqrt{\left[\frac{D^2-(r_2)^2+(r_1)^2}{2\,D}\right]^2 - (r_1)^2} = \sqrt{\frac{D^2}{4} - r^2} = 1\ (metre, yard, \dots)$$

The cylinder will have the following contour coordinate u_K (see I.2.2/3)

$$u_{K1} = \sinh^{-1}\left(\frac{a}{r_1}\right) = \sinh^{-1}\left(\frac{1}{0{,}8509}\right) = +1$$

$$u_{K2} = -\sinh^{-1}\left(\frac{a}{r_2}\right) = -\sinh^{-1}\left(\frac{1}{0{,}8509}\right) = -1$$

The live conductors are located at x_{L1} and $x_{L2} = -x_{L1}$ with $y_{L1,2} = 0$

$$v_{L1,2} = \tan^{-1}\frac{-2\,a\,y_{L1,2}}{(x_L)^2 + (y_{L1,2})^2 - a^2} = \tan^{-1} 0 = \begin{cases} \pi & for\ x < a \\ 0 & for\ x > a \end{cases}$$

Due to the fact that the conductors has to be outside the cylinder we will have two regions

a.) $0 < x < D/2 - r$ with $v_{L1,2} = \pi$ and u_1

b.) $D/2 + r < x < \infty$ with $v_{L1,2} = 0$ and u_1

herewith

$$u_{L1,2} = \tanh^{-1}\frac{2\,a\,x_{L1,2}}{(x_L)^2 + (y_L)^2 + a^2} = \tanh^{-1}\frac{2\,a\,x_{L1,2}}{(x_L)^2 + a^2}$$

a.) Force on conductor 1 within the region $0 < x < D/2 - r$

The coefficients of the different equations are in accordance with below mentioned equations.

I_1	$I_2 = -\Delta\, I_1$	u_1	$u_2 = -u_1$	$v_{1,2} = \pi$
u_{K1}	$u_{K2} = -u_{K1}$	contour coordinates of cylinder		$\mu_{1,2} = \infty$

II.1.5 Coefficients of exciting field with current I_1

$$A_p = -\frac{\mu_0 I_1}{2\pi}\frac{1}{p} \qquad B_p = \frac{\mu_0 I_1}{2\pi}\frac{(-1)^p}{p e^{p u_1}} = -A_p \frac{(-1)^p}{e^{p u_1}} \qquad C_p = \frac{\mu_0 I_1}{2\pi}\frac{\sin p v_1}{p e^{p u_1}} = 0$$

II.2.1 Coefficients of exciting field with current I_2

$$A_p^* = -\frac{\mu_0 I_2}{2\pi}\frac{1}{p} \qquad B_p^* = \frac{\mu_0 I_2}{2\pi}\frac{(-1)^p}{p e^{-p(-u_1)}} = -A_p^* \frac{(-1)^p}{e^{p u_1}} \qquad C_p^* = \frac{\mu_0 I_2}{2\pi}\frac{\sin p v_2}{p e^{-p u_2}} = 0$$

$$A_p^* = -\frac{\mu_0 (-\Delta I_1)}{2\pi}\frac{1}{p} = -\Delta A_p \qquad B_p^* = +\Delta A_p \frac{(-1)^p}{e^{p u_1}}$$

II.3.5 Coefficients of exciting field with two conductors with current $I_{1,2}$

$$D_{Ip} = A_p + B_p + A_p^* + B_p^* e^{-2pu_2} = A_p \left[(1-\Delta) - \frac{(-1)^p}{e^{p u_1}} + \Delta \frac{(-1)^p}{e^{p u_1}} e^{2pu_1}\right]$$

$$D_{Ip} = A_p \left[(1-\Delta) - \left(e^{-pu_1} - \Delta e^{pu_1}\right)(-1)^p\right]$$

$$D_{IVp} = A_p + B_p e^{2pu_1} + A_p^* + B_p^* = A_p \left[(1-\Delta) - \frac{(-1)^p}{e^{p u_1}} e^{2pu_1} + \Delta \frac{(-1)^p}{e^{p u_1}}\right]$$

$$D_{IVp} = A_p \left[(1-\Delta) + \left(\Delta e^{-pu_1} - e^{pu_1}\right)(-1)^p\right]$$

Note: for $\Delta=1$ is $D_{IVp} = -D_{Ip}$

II.3.6 Coefficients of exciting field with two conductors with current $I_{1,2}$

$$E_{Ip} = C_p + C_p^* e^{-2pu_2} = 0 \qquad E_{IVp} = C_p e^{2pu_1} + C_p^* = 0$$

II.4.6 Coefficients of "Stör" field with two conductors with current $I_{1,2}$ for high permeability

$$C_{pStör}F_{pStör} = \frac{D_{Ip} e^{p(u_{K1}+u_{K2})} + \left(\frac{\mu_1-\mu_0}{\mu_1+\mu_0}\right) D_{IVp} e^{-p(u_{K1}-u_{K2})}}{\left(\frac{\mu_2+\mu_0}{\mu_2-\mu_0}\right) e^{p(u_{K1}-u_{K2})} - \left(\frac{\mu_1-\mu_0}{\mu_1+\mu_0}\right) e^{-p(u_{K1}-u_{K2})}} = \frac{D_{Ip} + D_{IVp} e^{-2pu_{K1}}}{e^{2pu_{K1}} - e^{-2pu_{K1}}}$$

>> CF

II.4.7 Coefficients of "Stör" field with two conductors with current $I_{1,2}$

$$C_{pStör} E_{pStör} = \left(+D_{IVp} + C_{pStör}F_{pStör}\right) e^{-2pu_{K1}}$$

$$C_{pStör} E_{pStör} = \left(+D_{IVp} + \frac{D_{Ip} + D_{IVp} e^{-2pu_{K1}}}{e^{2pu_{K1}} - e^{-2pu_{K1}}}\right) e^{-2pu_{K1}}$$

$$C_{pStör} E_{pStör} = \left[\frac{D_{IVp}\left(e^{2pu_{K1}} - e^{-2pu_{K1}}\right)}{e^{2pu_{K1}} - e^{-2pu_{K1}}} + \frac{D_{Ip} + D_{IVp} e^{-2pu_{K1}}}{e^{2pu_{K1}} - e^{-2pu_{K1}}}\right] e^{-2pu_{K1}}$$

$$C_{pStör} E_{pStör} = \left[\frac{D_{IVp} e^{2pu_{K1}}}{e^{2pu_{K1}} - e^{-2pu_{K1}}} + \frac{D_{Ip}}{e^{2pu_{K1}} - e^{-2pu_{K1}}}\right] e^{-2pu_{K1}} = \frac{D_{IVp} + D_{Ip} e^{-2pu_{K1}}}{e^{2pu_{K1}} - e^{-2pu_{K1}}}$$

>> CE

II.4.10 Coefficients of "Stör" field with two conductors with current $I_{1,2}$

$$D_{pStör}F_{pStör} = \frac{E_{Ip} e^{p(u_{K1}+u_{K2})} + \left(\frac{\mu_1-\mu_0}{\mu_1+\mu_0}\right) E_{IVp} e^{-p(u_{K1}-u_{K2})}}{\left(\frac{\mu_2+\mu_0}{\mu_2-\mu_0}\right) e^{p(u_{K1}-u_{K2})} - \left(\frac{\mu_1-\mu_0}{\mu_1+\mu_0}\right) e^{-p(u_{K1}-u_{K2})}} = 0$$

because E = 0

II.4.11 Coefficients of "Stör" field with two conductors with current $I_{1,2}$

$$D_{pStör}E_{pStör} = \left(E_{IVp} + D_{pStör}F_{pStör}\right)\left(\frac{\mu_1-\mu_0}{\mu_1+\mu_0}\right) e^{-2pu_{K1}} = 0$$

because E and DF = 0

The force on conductor 1 is based on equation III.1.1 as follows

$$\frac{d\overrightarrow{K_1}}{dz} = \frac{I_1}{h(u_1,v_1)}\sum_{p=1}^{\infty} p \left\{ \begin{array}{c} \overrightarrow{e_u}[-(A_p^* + B_p^* + C_{pStör}F_{pStör})\cos pv_1 - (C_p^* + D_{pStör}F_{pStör})\sin pv_1]e^{-pu_1} \\ + \overrightarrow{e_u}[(C_{pStör}\,E_{pStör}\cos pv_1 + D_{pStör}\,E_{pStör}\sin pv_1)e^{pu_1}] \\ + \overrightarrow{e_v}[-(A_p^* + B_p^* + C_{pStör}F_{pStör})\sin pv_1 + (C_p^* + D_{pStör}F_{pStör})\cos pv_1]e^{-pu_1} \\ + \overrightarrow{e_v}[(-C_{pStör}\,E_{pStör}\sin pv_1 + D_{pStör}\,E_{pStör}\cos pv_1)e^{pu_1}] \end{array} \right\}$$

With $\cos pv_1 = (-1)^p$ and $\sin pv_1 = 0$, in addition C_p, C_p^*, $D_{pStör}E_{pStör}$, $D_{pStör}F_{pStör}$ are zero

We will get

$$\frac{d\overrightarrow{K_1}}{dz} = \frac{I_1}{h(u_1,v_1)}\sum_{p=1}^{\infty} p \left\{ \begin{array}{c} \overrightarrow{e_u}[-(A_p^* + B_p^* + C_{pStör}F_{pStör})\cos pv_1 - 000]e^{-pu_1} \\ + \overrightarrow{e_u}[(C_{pStör}\,E_{pStör}\cos pv_1 + 00)e^{pu_1}] \\ + \overrightarrow{e_v}[-(\ldots)0 + (00 + 00)\cos pv_1]e^{-pu_1} \\ + \overrightarrow{e_v}[(0 + 0\cos pv_1)e^{pu_1}] \end{array} \right\}$$

$$\frac{d\overrightarrow{K_1}}{dz} = \frac{I_1}{h(u_1,v_1)}\sum_{p=1}^{\infty} p\left\{\overrightarrow{e_u}\left[[-(A_p^* + B_p^* + C_{pStör}F_{pStör})(-1)^p]e^{-pu_1} + \left(C_{pStör}E_{pStör}(-1)^p\right)e^{pu_1}\right]\right\}$$

$$\frac{d\overrightarrow{K_1}}{dz} = -\overrightarrow{e_u}\frac{I_1}{h(u_1,v_1)}\sum_{p=1}^{\infty} p\,(-1)^p[(A_p^* + B_p^*)e^{-pu_1} + C_{pStör}F_{pStör}e^{-pu_1} - C_{pStör}E_{pStör}e^{pu_1}]$$

With metric coefficient h(u,v) I.5.1.we will get with I.6.3 and 4 the x/ y component for $v = \pi$

$$K_x = \frac{(1-\cosh u\cos v)\,K_u - \sinh u\sin v\,K_v}{\cosh u - \cos v} = \frac{[(1-(-1)\cosh u)K_u - \sinh u\sin\pi\,K_v]}{\cosh u + 1} = K_u$$

$$K_y = \frac{\sinh u\sin v\;K_u + (1-\cosh u\;\cos v)K_v}{\cosh u - \cos v} = \frac{[\sinh u\sin\pi\;K_u + (1-(-1)\cosh u)K_v]}{\cosh u + 1} = K_v$$

Therefore the force on conductor 1 within $0 < x < D/2 - r$

$$\boxed{\frac{d\overrightarrow{K_1}}{dz} = -\overrightarrow{e_x}\,I_1\frac{(\cosh u_1 + 1)}{a}\sum_{p=1}^{\infty} p\,(-1)^p[(A_p^* + B_p^*)e^{-pu_1} + C_{pStör}F_{pStör}e^{-pu_1} - C_{pStör}E_{pStör}e^{pu_1}]}$$

General case for $I_2 \neq I_1$ (III.2.1)

<u>Special Case</u>: $I_2 = -I_1$

$$A_p^* = -A_p \qquad B_p^* = +A_p\frac{(-1)^p}{e^{pu_1}} \qquad D_{Ip} = -A_p(e^{-pu_1} - e^{pu_1})(-1)^p \qquad D_{IVp} = -D_{Ip}$$

$$C_{pStör}F_{pStör} = \frac{D_{Ip} - D_{Ip}e^{-2pu_{K1}}}{e^{2pu_{K1}} - e^{-2pu_{K1}}} = \frac{D_{Ip}(1-e^{-2pu_{K1}})}{e^{2pu_{K1}} - e^{-2pu_{K1}}} = \frac{-A_p(e^{-pu_1} - e^{pu_1})(-1)^p\,(1-e^{-2pu_{K1}})}{e^{2pu_{K1}} - e^{-2pu_{K1}}}$$

$$C_{pStör}E_{pStör} = \frac{-D_{Ip} + D_{Ip}e^{-2pu_{K1}}}{e^{2pu_{K1}} - e^{-2pu_{K1}}} = \frac{-D_{Ip}\,(1-e^{-2pu_{K1}})}{e^{2pu_{K1}} - e^{-2pu_{K1}}} = -C_{pStör}F_{pStör}$$

$$\frac{d\overrightarrow{K_1}}{dz} = -\overrightarrow{e_x}\frac{I_1(\cosh u_1 + 1)}{a}\sum_{p=1}^{\infty} p\,(-1)^p\left\{\left(-A_p + A_p\frac{(-1)^p}{e^{pu_1}}\right)e^{-pu_1} + C_{pStör}F_{pStör}(e^{-pu_1} + e^{pu_1})\right\}$$

$$\frac{d\overrightarrow{K_1}}{dz} = -\overrightarrow{e_x}\frac{I_1(\cosh u_1 + 1)}{a}\sum_{p=1}^{\infty} p(-A_p)\left\{ \begin{array}{c} ((-1)^p - e^{-pu_1})e^{-pu_1} + \\ \frac{(e^{-pu_1} - e^{pu_1})\,(1-e^{-2pu_{K1}})}{e^{2pu_{K1}} - e^{-2pu_{K1}}}(e^{-pu_1} + e^{pu_1}) \end{array} \right\}$$

$$\boxed{\frac{d\overrightarrow{K_1}}{dz} = -\overrightarrow{e_x}\,I_1\frac{\mu_0\,I_1}{2\,\pi}\frac{(\cosh u_1 + 1)}{a}\sum_{p=1}^{\infty}\left\{[(-1)^p - e^{-pu_1}]e^{-pu_1} + \frac{(1-e^{-2pu_{K1}})(e^{-2pu_1} - e^{2pu_1})}{e^{2pu_{K1}} - e^{-2pu_{K1}}}\right\}}$$

Special case for $I_2 = -I_1$ (III.2.2)

b.) Force on conductor 1 within the region $D/2 + r < x < \infty$

The coefficients of the different equations are in accordance with below mentioned equations.

I_1	$I_2 = -\Delta\, I_1$	u_1	$u_2 = -u_1$	$v_{1,2} = 0$
u_{K1}	$u_{K2} = -u_{K1}$	contour coordinates of cylinder		$\mu_{1,2} = \infty$

II.1.5 Coefficients of exciting field with current I_1

$$A_p = -\frac{\mu_0\, I_1}{2\,\pi}\frac{1}{p} \qquad B_p = \frac{\mu_0 I_1}{2\pi}\frac{1}{p e^{pu_1}} = -A_p\frac{1}{e^{pu_1}} \qquad C_p = \frac{\mu_0 I_1}{2\pi}\frac{\sin pv_1}{p e^{pu_1}} = 0$$

II.2.1 Coefficients of exciting field with current I_2

$$A_p^* = -\frac{\mu_0\, I_2}{2\,\pi}\frac{1}{p} \qquad B_p^* = \frac{\mu_0 I_2}{2\pi}\frac{1}{p e^{-p(-u_1)}} = -A_p^*\frac{1}{e^{pu_1}} \qquad C_p^* = \frac{\mu_0 I_2}{2\pi}\frac{\sin pv_2}{p e^{-pu_2}} = 0$$

$$A_p^* = -\frac{\mu_0\,(-\Delta I_1)}{2\,\pi}\frac{1}{p} = -\Delta A_p \qquad B_p^* = +\Delta A_p\frac{1}{e^{pu_1}}$$

II.3.5 Coefficients of exciting field with two conductors with current $I_{1,2}$

$$D_{Ip} = A_p + B_p + A_p^* + B_p^* e^{-2pu_2} = A_p\left[(1-\Delta) - \frac{1}{e^{pu_1}} + \Delta A_p\frac{1}{e^{pu_1}}e^{2pu_1}\right]$$

$$D_{Ip} = A_p\,[(1-\Delta) - (e^{-pu_1} - \Delta e^{pu_1})]$$

$$D_{IVp} = A_p + B_p e^{2pu_1} + A_p^* + B_p^* = A_p\left[(1-\Delta) - \frac{1}{e^{pu_1}}e^{2pu_1} + \Delta\frac{1}{e^{pu_1}}\right]$$

$$D_{IVp} = A_p\,[(1-\Delta) + (\Delta e^{-pu_1} - e^{pu_1})]$$

Note: for $\Delta=1$ is $D_{IVp} = -D_{Ip}$

The other coefficient has the same structure as before

II.4.6 Coefficients of "Stör" field with two conductors with current $I_{1,2}$ for high permeability

$$C_{pStör}F_{pStör} = \frac{D_{Ip} + D_{IVp}e^{-2pu_{K1}}}{e^{2pu_{K1}} - e^{-2pu_{K1}}}$$ >> **CF**

II.4.7 Coefficients of "Stör" field with two conductors with current $I_{1,2}$

$$C_{pStör}\,E_{pStör} = \frac{D_{IVp} + D_{Ip}e^{-2pu_{K1}}}{e^{2pu_{K1}} - e^{-2pu_{K1}}}$$ >> **CE**

II.4.10 Coefficients of "Stör" field with two conductors with current $I_{1,2}$

$$D_{pStör}F_{pStör} = 0$$ because $E = 0$

II.4.11 Coefficients of "Stör" field with two conductors with current $I_{1,2}$

$$D_{pStör}\,E_{pStör} = 0$$ because E and DF = 0

The force on conductor 1 is based on equation III.1.1 as follows

$$\frac{\overrightarrow{dK_1}}{dz} = \frac{I_1}{h(u_1,v_1)}\sum_{p=1}^{\infty} p \left\{\begin{array}{c} \overrightarrow{e_u}\left[-\left(A_p^* + B_p^* + C_{pStör}F_{pStör}\right)\cos pv_1 - \left(C_p^* + D_{pStör}F_{pStör}\right)\sin pv_1\right]e^{-pu_1} \\ + \overrightarrow{e_u}\left[\left(C_{pStör}\,E_{pStör}\cos pv_1 + D_{pStör}\,E_{pStör}\sin pv_1\right)e^{pu_1}\right] \\ + \overrightarrow{e_v}\left[-\left(A_p^* + B_p^* + C_{pStör}F_{pStör}\right)\sin pv_1 + \left(C_p^* + D_{pStör}F_{pStör}\right)\cos pv_1\right]e^{-pu_1} \\ + \overrightarrow{e_v}\left[\left(-C_{pStör}\,E_{pStör}\sin pv_1 + D_{pStör}\,E_{pStör}\cos pv_1\right)e^{pu_1}\right] \end{array}\right\}$$

With $\cos pv_1 = 1$ and $\sin pv_1 = 0$, in addition C_p, C_p*, $D_{pStör}E_{pStör}$, $D_{pStör}F_{pStör}$ are zero

We will get

$$\frac{\overrightarrow{dK_1}}{dz} = \frac{I_1}{h(u_1,v_1)}\sum_{p=1}^{\infty} p \begin{Bmatrix} \overrightarrow{e_u}[-(A_p^* + B_p^* + C_{pStör}F_{pStör})\cos pv_1 - 000]e^{-pu_1} \\ +\overrightarrow{e_u}[(C_{pStör}\, E_{pStör}\cos pv_1 + 00)e^{pu_1}] \\ +\overrightarrow{e_v}[-(\ldots)0 + (00+00)\cos pv_1]e^{-pu_1} \\ +\overrightarrow{e_v}[(0 + 0\cos pv_1)e^{pu_1}] \end{Bmatrix}$$

$$\frac{\overrightarrow{dK_1}}{dz} = \frac{I_1}{h(u_1,v_1)}\sum_{p=1}^{\infty} p\left\{\overrightarrow{e_u}\left[\left[-(A_p^* + B_p^* + C_{pStör}F_{pStör})\right]e^{-pu_1} + (C_{pStör}E_{pStör})e^{pu_1}\right]\right\}$$

$$\frac{\overrightarrow{dK_1}}{dz} = -\overrightarrow{e_u}\frac{I_1}{h(u_1,v_1)}\sum_{p=1}^{\infty} p\left[(A_p^* + B_p^*)e^{-pu_1} + C_{pStör}F_{pStör}e^{-pu_1} - C_{pStör}E_{pStör}e^{pu_1}\right]$$

With metric coefficient h(u,v) I.5.1 we will get with I.6.3 and 4 the x/ y component especially for **v = 0**

$$K_x = \frac{(1-\cosh u\cos v)\,K_u - \sinh u\sin v\,K_v}{\cosh u - \cos v} = \frac{(1-\cosh u\cos 0)\,K_u - \sinh u\sin 0\,K_v}{\cosh u - 1} = -K_u$$

$$K_y = \frac{\sinh u\sin v\;K_u + (1-\cosh u\;\cos v)K_v}{\cosh u - cos v} = \frac{\sinh u\sin 0\;K_u + (1-\cosh u\;\cos 0)K_v}{\cosh u + 1} = -K_v$$

Therefore the force on conductor 1 within D/2 + r < x < ∞

$$\boxed{\frac{\overrightarrow{dK_1}}{dz} = +\overrightarrow{e_x}\,I_1\frac{(\cosh u_1 - 1)}{a}\sum_{p=1}^{\infty} p\left[(A_p^* + B_p^*)e^{-pu_1} + C_{pStör}F_{pStör}e^{-pu_1} - C_{pStör}E_{pStör}e^{pu_1}\right]}$$

General case for $I_2 \neq I_1$ (III.2.3)

Special Case: $I_2 = -I_1$

$$A_p^* = -A_p \qquad B_p^* = +A_p e^{-pu_1} \qquad D_{Ip} = -A_p(e^{-pu_1} - e^{pu_1}) \qquad D_{IVp} = -D_{Ip}$$

$$C_{pStör}F_{pStör} = \frac{D_{Ip} - D_{Ip}e^{-2pu_{K1}}}{e^{2pu_{K1}} - e^{-2pu_{K1}}} = \frac{D_{Ip}(1-e^{-2pu_{K1}})}{e^{2pu_{K1}} - e^{-2pu_{K1}}} = \frac{-A_p(e^{-pu_1} - e^{pu_1})\,(1-e^{-2pu_{K1}})}{e^{2pu_{K1}} - e^{-2pu_{K1}}}$$

$$C_{pStör}E_{pStör} = \frac{-D_{Ip} + D_{Ip}e^{-2pu_{K1}}}{e^{2pu_{K1}} - e^{-2pu_{K1}}} = \frac{-D_{Ip}\,(1-e^{-2pu_{K1}})}{e^{2pu_{K1}} - e^{-2pu_{K1}}} = -C_{pStör}F_{pStör}$$

$$\frac{\overrightarrow{dK_1}}{dz} = +\overrightarrow{e_x}\,I_1\frac{(\cosh u_1 - 1)}{a}\sum_{p=1}^{\infty} p\left[(-A_p + A_p e^{-pu_1})e^{-pu_1} + C_{pStör}F_{pStör}(e^{-pu_1} + e^{pu_1})\right]$$

$$\frac{\overrightarrow{dK_1}}{dz} = +\overrightarrow{e_x}\,I_1\frac{(\cosh u_1 - 1)}{a}\sum_{p=1}^{\infty} p(-A_p)\left\{[1 - e^{-pu_1}]e^{-pu_1} + \frac{(e^{-pu_1} - e^{pu_1})\,(1-e^{-2pu_{K1}})}{e^{2pu_{K1}} - e^{-2pu_{K1}}}(e^{-pu_1} + e^{pu_1})\right\}$$

$$\boxed{\frac{\overrightarrow{dK_1}}{dz} = +\overrightarrow{e_x}\,I_1\frac{\mu_0\,I_1}{2\,\pi}\frac{(\cosh u_1 - 1)}{a}\sum_{p=1}^{\infty}\left\{[1 - e^{-pu_1}]e^{-pu_1} + \frac{(1-e^{-2pu_{K1}})(e^{-2pu_1} - e^{2pu_1})}{e^{2pu_{K1}} - e^{-2pu_{K1}}}\right\}}$$

Special case for $I_2 = -I_1$ (III.2.4)

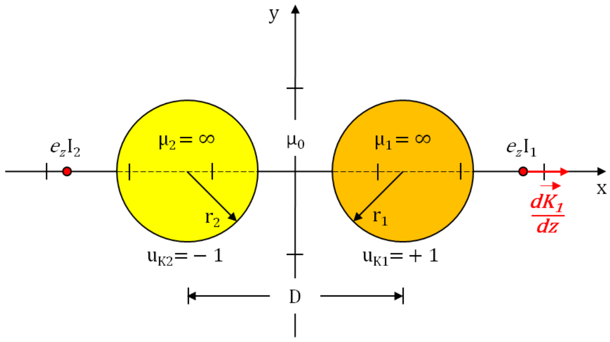
Test:

x = ∞ u = 0 therefore the sum of [1-e^0] and (e^{-0}-e^0) will be also zero.

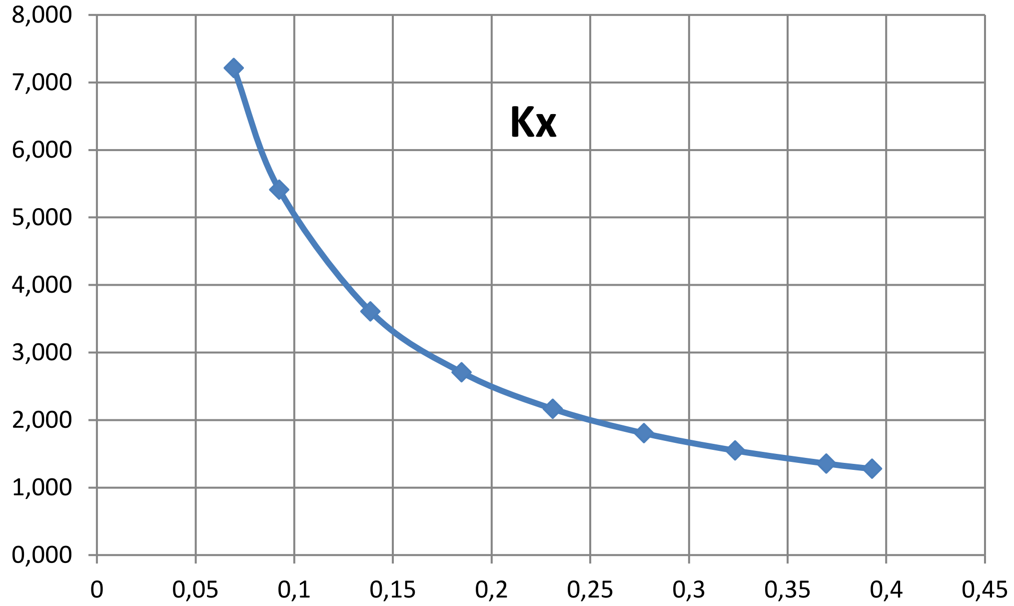
x = D/2+r u = u_{K1} therefore the sum of {[…] + (1-$e^{-\ldots}$)(-1)} will be negative infinitive

Example 1: One pair conductors without cylinder > permeability = 1,001

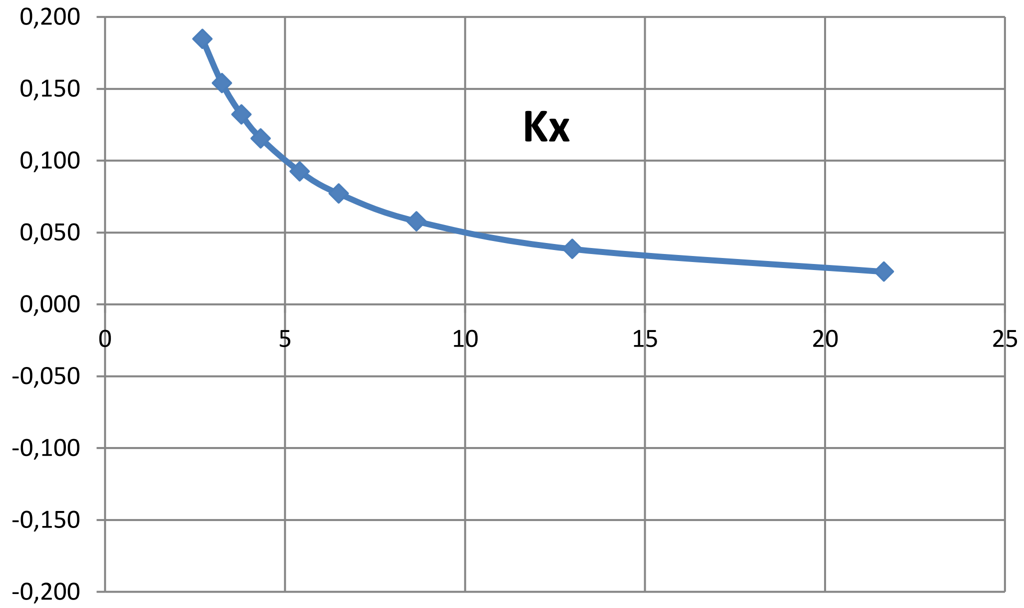
Körper 1		Abstand		Körper 2
Radius_1	0,8509	2,626	0,8509	Radius_2
Per_mü1	1,001		1,001	Per_mü2
Leiter 1				**Leiter 2**
Strom_1	1		-1	Strom_2
L-ort x1	variabel		- - -	L-ort x2
L-ort y1	0		0	L-ort y2



Force on conductor 1 between cylinders on x-axis – **all forces normalized to** $\frac{\mu_0}{2\,\pi}$



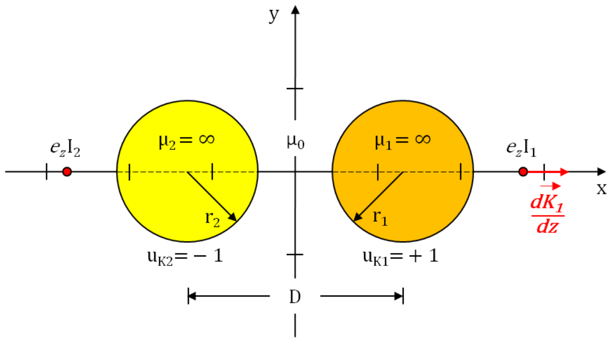
Force on conductor 1 on positive x-axis



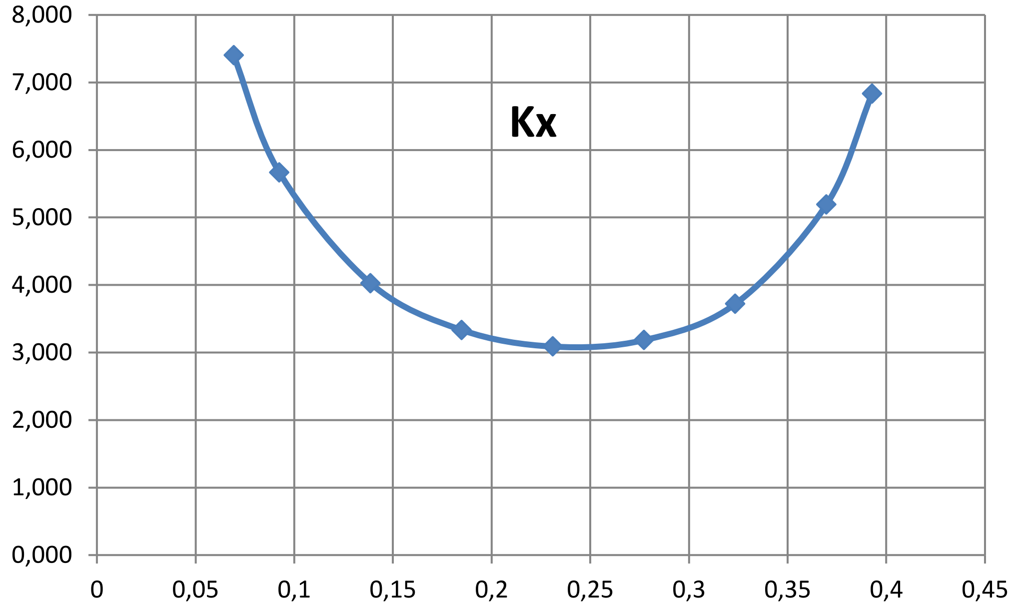
Conductor one with positive force in x-axis >>> continual rejection

Example 2: One pair of conductors with forward and return current

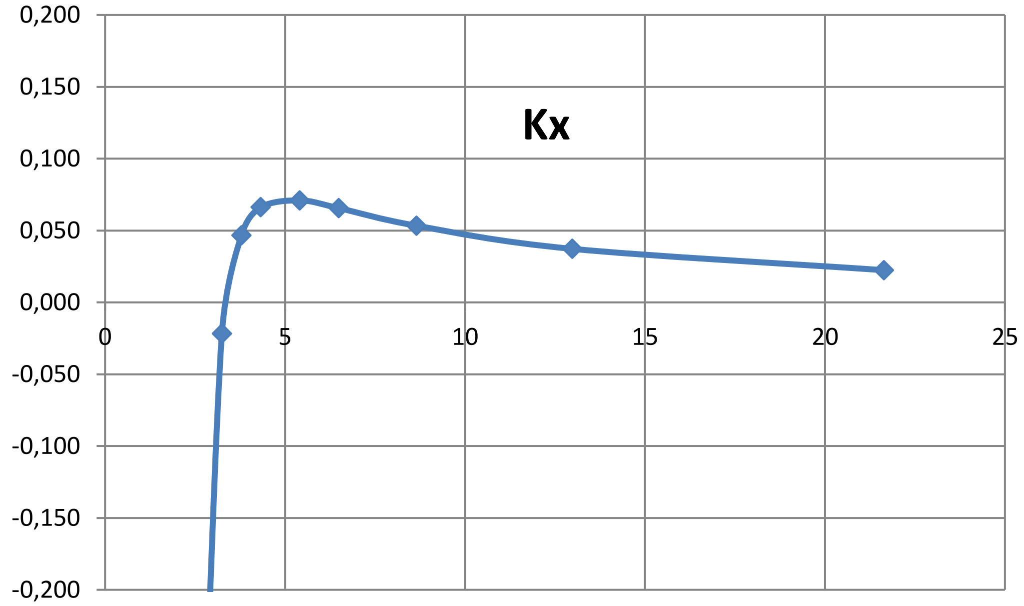
Körper 1		Abstand		**Körper 2**
Radius_1	0,8509	2,626	0,8509	Radius_2
Per_mü1	999		999	Per_mü2
Leiter 1				**Leiter 2**
Strom_1	1		-1	Strom_2
L-ort x1	variabel		- - -	L-ort x2
L-ort y1	0		0	L-ort y2



Force on conductor 1 between cylinders on x-axis - **all forces normalized to** $\frac{\mu_0}{2\,\pi}$



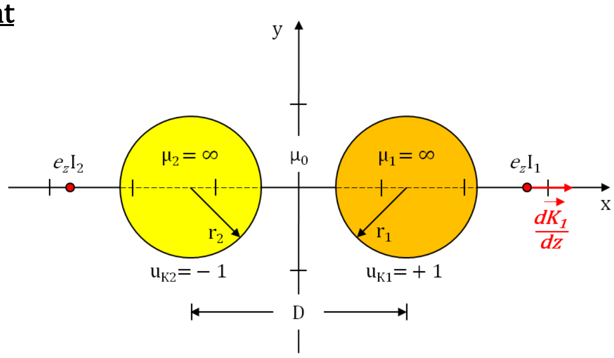
Force on conductor 1 on positive x-axis



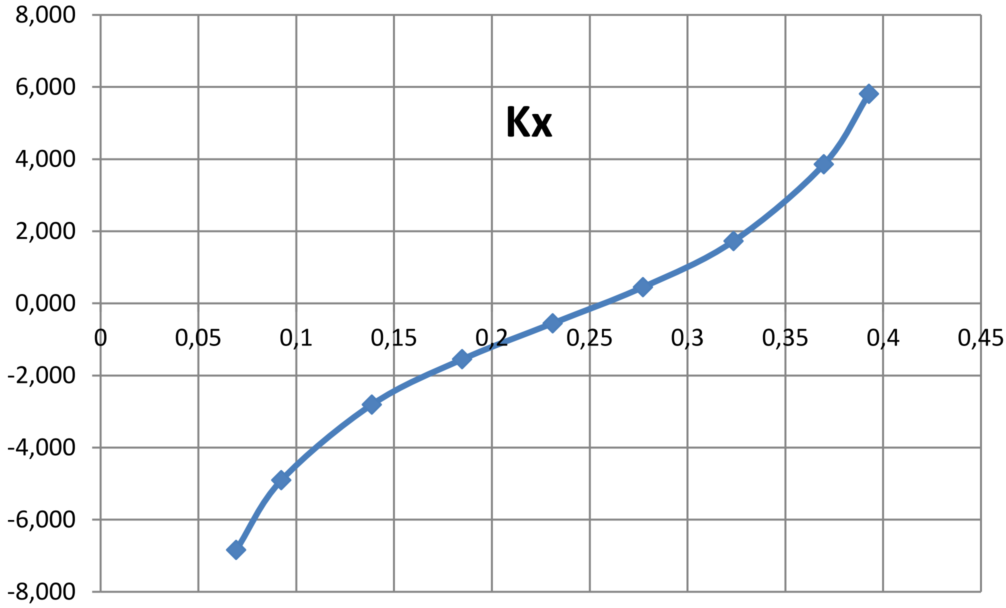
Conductor one with minimum force between cylinder

Example 3: Two conductors with forward current

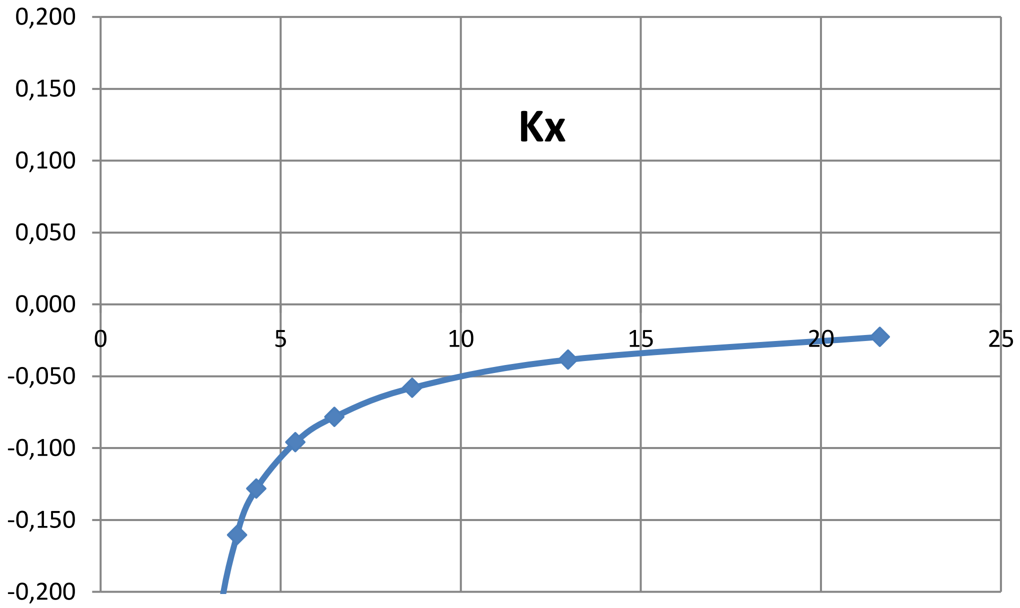
Körper 1		Abstand		**Körper 2**
Radius_1	0,8509	2,626	0,8509	Radius_2
Per_mü1	999		999	Per_mü2
Leiter 1				**Leiter 2**
Strom_1	1		1	Strom_2
L-ort x1	variabel		- - -	L-ort x2
L-ort y1	0		0	L-ort y2



Force on conductor 1 between cylinders on x-axis – **all forces normalized to** $\frac{\mu_0}{2\,\pi}$



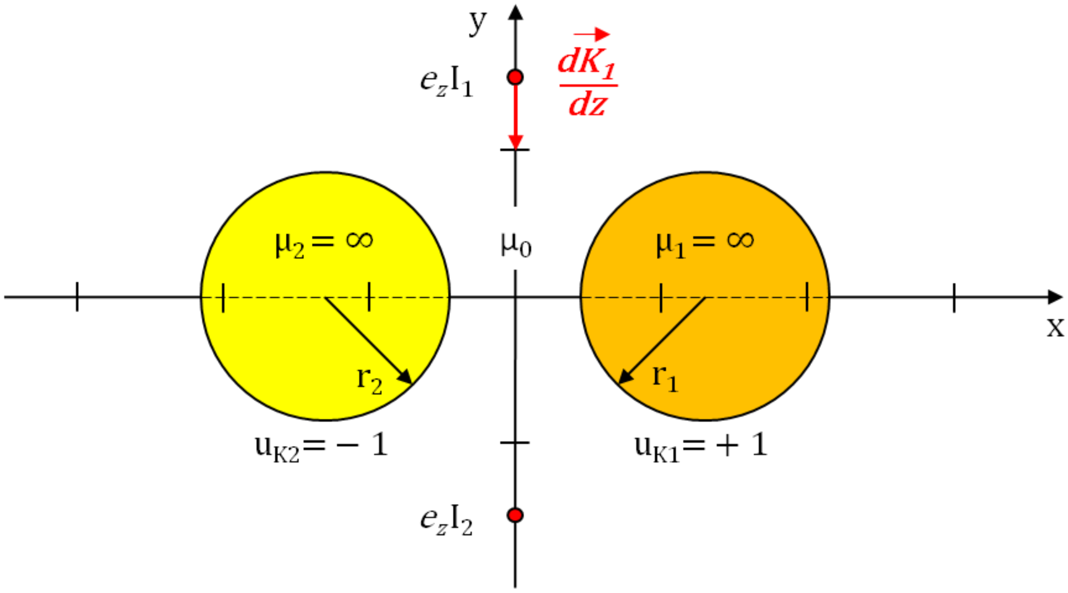
Force on conductor 1 on positive x-axis



Conductor one with no force between cylinder, always attraction

III.3 Example: Force on two conductors on y-axis

The live conductors with current I will be placed on the y-axis.



The geometric situation is in all other aspects the same as before in the previous example.

D = 2,626 (metre, yard, ...) r = 0,8509 (metre, yard, ...)

The geometric constant a of the bipolar-cylindrical coordinates will be (see I.2.1)

$$a = \sqrt{\left[\frac{D^2-(r_2)^2+(r_1)^2}{2\,D}\right]^2 - (r_1)^2} = \sqrt{\frac{D^2}{4} - r^2} = 1\ (metre, yard, \dots)$$

The cylinder will have the following contour coordinate u_K (see I.2.2/3)

$$u_{K1} = \sinh^{-1}\left(\frac{a}{r_1}\right) = \sinh^{-1}\left(\frac{1}{0{,}8509}\right) = +1$$

$$u_{K2} = -\sinh^{-1}\left(\frac{a}{r_2}\right) = -\sinh^{-1}\left(\frac{1}{0{,}8509}\right) = -1$$

The live conductors are located at y_{L1} and $y_{L2} = -y_{L1}$ with $x_{L1,2} = 0$

$$u_{L1} = \tanh^{-1}\frac{2\,a\,x_{L1}}{(x_{L1})^2+(y_{L1})^2+a^2} = \tanh^{-1}\frac{0}{(y_{L1})^2+a^2} = 0$$

$$v_{L1} = \tan^{-1}\frac{-2\,a\,y_{L1}}{(x_{L1})^2+(y_{L1})^2-a^2} = \tan^{-1}\frac{-2\,a\,y_{L1}}{(y_{L1})^2-a^2} = -\tan^{-1}\frac{2\,a\,y_{L1}}{(y_{L1})^2-a^2}$$

For $y = 0$ the v-coordinate will be also 0

For $y = a$ the v-coordinate will be $\pi/2$

For $y = \infty$ the v-coordinate will be 0

The metric factor h(u=0, v) will be $h = \frac{a}{\cosh u - \cos v} = \frac{a}{\cosh 0 - \cos v} = \frac{a}{1 - \cos v}$

Force on conductor 1 within the region $0 < y < \infty$

The coefficients of the different equations are in accordance with below mentioned equations.

I_1	$I_2 = -\Delta\, I_1$	v_1	$v_2 = -v_1$	$u_{1,2} = 0$
u_{K1}	$u_{K2} = -u_{K1}$	contour coordinates of cylinder		$\mu_{1,2} = \infty$

II.1.5 Coefficients of exciting field with current I_1

$$A_p = -\frac{\mu_0 I_1}{2\pi}\frac{1}{p} \qquad B_p = \frac{\mu_0 I_1}{2\pi}\frac{\cos pv_1}{pe^{p0}} = -A_p \cos pv_1 \qquad C_p = \frac{\mu_0 I_1}{2\pi}\frac{\sin pv_1}{pe^{p0}} = -A_p \sin pv_1$$

II.2.1 Coefficients of exciting field with current I_2 with cos(x)=cos(−x) and sin(−x)= −sin(x)

$$A_p^* = -\frac{\mu_0 I_2}{2\pi}\frac{1}{p} \qquad B_p^* = \frac{\mu_0 I_2}{2\pi}\frac{\cos pv_2}{pe^{-p0}} = -A_p^* \cos pv_1 \qquad C_p^* = \frac{\mu_0 I_2}{2\pi}\frac{\sin pv_2}{pe^{-p0}} = +A_p^* \sin pv_1$$

$$A_p^* = -\frac{\mu_0 (-\Delta I_1)}{2\pi}\frac{1}{p} = -\Delta A_p \qquad B_p^* = +\Delta A_p \cos pv_1 \qquad C_p^* = -\Delta A_p \sin pv_1$$

II.3.5 Coefficients of exciting field with two conductors with current $I_{1,2}$ for u=0

$$D_{Ip} = A_p + B_p + A_p^* + B_p^* e^{-2pu_2} = A_p\,[(1-\Delta) + (-\cos pv_1 + \Delta \cos pv_1\, e^{-2pu_2})]$$

$$D_{Ip} = A_p\,[(1-\Delta) - \cos pv_1\,(1-\Delta)] = A_p[(1-\Delta) + \cos pv_1\,(-1+\Delta e^{-2pu_2})]$$

$$D_{IVp} = A_p + B_p e^{2pu_1} + A_p^* + B_p^* = A_p[(1-\Delta) + \cos pv_1\,(-e^{2pu_1} + \Delta)]$$

Note: for equal currents $I_2 = -I_1$ (for Δ=1) on x-axis with u=0 both coefficients D_i will be zero

II.3.6 Coefficients of exciting field with two conductors with current $I_{1,2}$ for u=0

$$E_{Ip} = C_p + C_p^* e^{-2pu_2} = -A_p \sin pv_1 - \Delta A_p \sin pv_1 = -A_p(1+\Delta)\sin pv_1$$

$$E_{IVp} = C_p e^{2pu_1} + C_p^* = -A_p \sin pv_1 - \Delta A_p \sin pv_1 = -A_p(1+\Delta)\sin pv_1 = E_{Ip}$$

II.4.6 Coefficients of "Stör" field with two conductors with current $I_{1,2}$ for high permeability

$$C_{pSt\ddot{o}r}F_{pSt\ddot{o}r} = \frac{D_{Ip}+D_{IVp}e^{-2pu_{K1}}}{e^{2pu_{K1}} - e^{-2pu_{K1}}} = 0 \qquad \text{for } u = 0 \qquad >>$$

II.4.7 Coefficients of "Stör" field with two conductors with current $I_{1,2}$

$$C_{pSt\ddot{o}r}E_{pSt\ddot{o}r} = \left(D_{IVp} + C_{pSt\ddot{o}r}F_{pSt\ddot{o}r}\right)e^{-2pu_{K1}} = 0 \qquad \text{for } u = 0 \qquad >>$$

Note: for equal currents $I_2 = -I_1$ (for Δ=1) both coefficients CEF_i will be zero

II.4.10 Coefficients of "Stör" field with two conductors with current $I_{1,2}$

$$D_{pSt\ddot{o}r}F_{pSt\ddot{o}r} = \frac{E_{Ip}e^{p(u_{K1}+u_{K2})} + \left(\frac{\mu_1-\mu_0}{\mu_1+\mu_0}\right)E_{IVp}e^{-p(u_{K1}-u_{K2})}}{\left(\frac{\mu_2+\mu_0}{\mu_2-\mu_0}\right)e^{p(u_{K1}-u_{K2})} - \left(\frac{\mu_1-\mu_0}{\mu_1+\mu_0}\right)e^{-p(u_{K1}-u_{K2})}} = \frac{E_{Ip}(1+e^{-2pu_{K1}})}{e^{2pu_{K1}} - e^{-2pu_{K1}}}$$

$$D_{pSt\ddot{o}r}F_{pSt\ddot{o}r} = \frac{-A_p(1+\Delta)\sin pv_1(1+e^{-2pu_{K1}})}{e^{2pu_{K1}} - e^{-2pu_{K1}}} \qquad >>$$

II.4.11 Coefficients of "Stör" field with two conductors with current $I_{1,2}$

$$D_{pSt\ddot{o}r}E_{pSt\ddot{o}r} = \left(E_{IVp} + D_{pSt\ddot{o}r}F_{pSt\ddot{o}r}\right)\left(\frac{\mu_1-\mu_0}{\mu_1+\mu_0}\right)e^{-2pu_{K1}} = \left(E_{Ip} + \frac{E_{Ip}(1+e^{-2pu_{K1}})}{e^{2pu_{K1}} - e^{-2pu_{K1}}}\right)e^{-2pu_{K1}}$$

$$D_{pSt\ddot{o}r}E_{pSt\ddot{o}r} = E_{Ip}\left[\frac{e^{2pu_{K1}} - e^{-2pu_{K1}}}{e^{2pu_{K1}} - e^{-2pu_{K1}}} + \frac{1+e^{-2pu_{K1}}}{e^{2pu_{K1}} - e^{-2pu_{K1}}}\right]e^{-2pu_{K1}}$$

$$D_{pSt\ddot{o}r}E_{pSt\ddot{o}r} = \frac{-A_p(1+\Delta)\sin pv_1(1+e^{2pu_{K1}})}{e^{2pu_{K1}} - e^{-2pu_{K1}}}e^{-2pu_{K1}} = \frac{-A_p(1+\Delta)\sin pv_1(1+e^{-2pu_{K1}})}{e^{2pu_{K1}} - e^{-2pu_{K1}}} = D_{pSt\ddot{o}r}F_{pSt\ddot{o}r} >>$$

With metric coefficient h(u,v) I.5.1 we will get with I.6.3 and 4 the x/ y component for **u = 0**

$$K_x = \frac{(1-\cosh u \cos v)\,K_u - \sinh u \sin v\,K_v}{\cosh u - \cos v} = \frac{(1-\cosh 0 \cos v)\,K_u - \sinh 0 \sin v\,K_v}{\cosh 0 - \cos v} = +K_u$$

$$K_y = \frac{\sinh u \sin v\;K_u + (1-\cosh u\;\cos v)K_v}{\cosh u - cosv} = \frac{\sinh 0 \sin v\;K_u + (1-\cosh 0\;\cos v)K_v}{\cosh 0 - cosv} = +K_v$$

The force on conductor 1 is based on equation III.1.1 as follows

$$\frac{\overrightarrow{dK_1}}{dz} = \frac{I_1}{h(0,v_1)}\sum_{p=1}^{\infty} p \begin{Bmatrix} \overrightarrow{e_u}\left[-\left(A_p^* + B_p^* + C_{pStör}F_{pStör}\right)\cos pv_1 - \left(C_p^* + D_{pStör}F_{pStör}\right)\sin pv_1\right]e^{-pu_1} \\ + \overrightarrow{e_u}\left[\left(C_{pStör}\,E_{pStör}\cos pv_1 + D_{pStör}\,E_{pStör}\sin pv_1\right)e^{pu_1}\right] \\ + \overrightarrow{e_v}\left[-\left(A_p^* + B_p^* + C_{pStör}F_{pStör}\right)\sin pv_1 + \left(C_p^* + D_{pStör}F_{pStör}\right)\cos pv_1\right]e^{-pu_1} \\ + \overrightarrow{e_v}\left[\left(-C_{pStör}\,E_{pStör}\sin pv_1 + D_{pStör}\,E_{pStör}\cos pv_1\right)e^{pu_1}\right] \end{Bmatrix}$$

All exponential function in u will be for u=0 equal one, and with CF=CE, DF=DE therefore

$$\frac{\overrightarrow{dK_1}}{dz} = \frac{I_1}{h(0,v_1)}\sum_{p=1}^{\infty} p \begin{Bmatrix} \overrightarrow{e_u}\left[-\left(A_p^* + B_p^* + C_{pStör}F_{pStör}\right)\cos pv_1 - \left(C_p^* + D_{pStör}F_{pStör}\right)\sin pv_1\right] \\ + \overrightarrow{e_u}\left[\left(C_{pStör}\,E_{pStör}\cos pv_1 + D_{pStör}\,E_{pStör}\sin pv_1\right)\right] \\ + \overrightarrow{e_v}\left[-\left(A_p^* + B_p^* + C_{pStör}F_{pStör}\right)\sin pv_1 + \left(C_p^* + D_{pStör}F_{pStör}\right)\cos pv_1\right] \\ + \overrightarrow{e_v}\left[\left(-C_{pStör}\,E_{pStör}\sin pv_1 + D_{pStör}\,E_{pStör}\cos pv_1\right)\right] \end{Bmatrix}$$

$$\frac{\overrightarrow{dK_1}}{dz} = \frac{I_1}{h(0,v_1)}\sum_{p=1}^{\infty} p \begin{Bmatrix} \overrightarrow{e_u}\left[-\left(A_p^* + B_p^*\right)\cos pv_1 - C_p^*\sin pv_1\right] + \\ + \overrightarrow{e_v}\left[-\left(A_p^* + B_p^* + 2C_{pStör}F_{pStör}\right)\sin pv_1 + \left(C_p^* + 2D_{pStör}F_{pStör}\right)\cos pv_1\right] \end{Bmatrix}$$

The vector component in u-direction will be

$$[.u.] = -\left(A_p^* + B_p^*\right)\cos pv_1 - C_p^*\sin pv_1 = -A_p^*\cos pv_1 - B_p^*\cos pv_1 - C_p^*\sin pv_1$$

$$[.u.] = -\left[A_p^*\cos pv_1 + \left(-A_p^*\cos pv_1\right)\cos pv_1 + \left(A_p^*\sin pv_1\right)\sin pv_1\right]$$

$$[.u.] = -A_p^*\left[\cos pv_1 - \cos^2 pv_1 + \sin^2 pv_1\right] = -A_p^*\left[\cos pv_1 - \cos^2 pv_1 + \left(1 - cos^2\, pv_1\right)\right]$$

$$[.u.] = -A_p^*\left(1 + \cos pv_1 - 2\,cos^2 pv_1\right) = -\Delta\,\frac{\mu_0\, I_1}{2\,\pi}\,\frac{1+\cos pv_1 - 2cos^2 pv_1}{p}$$

<u>IMPORTANT</u>: Due to the fact that we are summing up here on the y-axis (with x=0 and therefore all exponential functions will be zero) – the convergence of this series is bad, especially for all x-values near x=0. We have to use another field ansatz for the case near y-axis!

The vector component in v-direction will be

$$[.v.] = -\left(A_p^* + B_p^* + 2C_{pStör}F_{pStör}\right)\sin pv_1 + \left(C_p^* + 2D_{pStör}F_{pStör}\right)\cos pv_1$$

$$[.v.] = -\left(A_p^* + B_p^*\right)\sin pv_1 + C_p^*\cos pv_1 + 2\,\frac{A_p(1-\Delta)(1-\cos pv_1)\,(1+e^{-2pu_{K1}})}{e^{2pu_{K1}} - e^{-2pu_{K1}}}\sin pv_1 + 2\,\frac{-A_p(1+\Delta)\sin pv_1(1+e^{-2pu_{K1}})}{e^{2pu_{K1}} - e^{-2pu_{K1}}}\cos pv_1$$

$$[.v.] = -\left(A_p^* + B_p^*\right)\sin pv_1 + C_p^*\cos pv_1 + 2\,\frac{A_p(1-\Delta)(1-\cos pv_1)\sin pv_1\,(1+e^{-2pu_{K1}})}{e^{2pu_{K1}} - e^{-2pu_{K1}}} + 2\,\frac{-A_p(1+\Delta)\sin pv_1\cos pv_1(1+e^{-2pu_{K1}})}{e^{2pu_{K1}} - e^{-2pu_{K1}}}$$

The first two elements 1 and 2

$$[12] = -\left(A_p^* + B_p^*\right)\sin pv_1 + C_p^*\cos pv_1 = -A_p^*\sin pv_1 - B_p^*\sin pv_1 + C_p^*\cos pv_1$$

$$[12] = -A_p^*\sin pv_1 + \left(+A_p^*\cos pv_1\right)\sin pv_1 + A_p^*\sin pv_1\cos pv_1$$

$$[12] = +\Delta A_p\left[\sin pv_1 - 2\sin pv_1\cos pv_1\right] = +\Delta A_p\sin pv_1\left(1 - \cos pv_1\right)$$

The second two elements 3 and 4 with exponential functions

$$[34] = 2\,\frac{A_p\left(1+e^{-2pu_{K1}}\right)}{e^{2pu_{K1}} - e^{-2pu_{K1}}}\left[(1-\Delta)(1-\cos pv_1)\sin pv_1 - (1+\Delta)\sin pv_1\cos pv_1\right]$$

$$[\ldots] = \left(1 - \cos pv_1 - \Delta + \Delta\cos pv_1\right)\sin pv_1 - \sin pv_1\cos pv_1 - \Delta\sin pv_1\cos pv_1$$

$$[\ldots] = \sin pv_1 - 2\cos pv_1\sin pv_1 - \Delta\sin pv_1 = \sin pv_1\left(1 - \Delta - 2\cos pv_1\right)$$

Therefore the 3&4 element

$$[34] = 2\,\frac{A_p\left(1+e^{-2pu_{K1}}\right)}{e^{2pu_{K1}} - e^{-2pu_{K1}}}\sin pv_1(1-\Delta-2\cos pv_1)$$

Finally we get the vector component in v-direction as follow

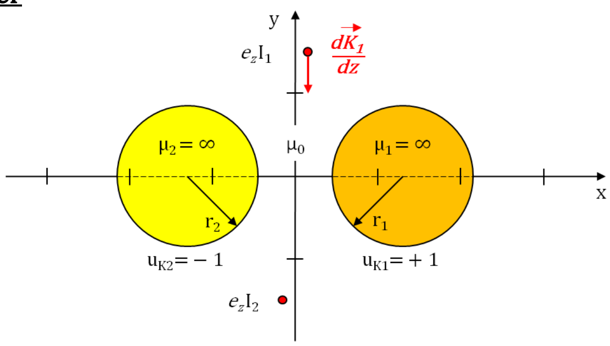
$$[.v.] = +\Delta A_p \sin pv_1\,(1-\cos pv_1) + \;2\,\frac{A_p\left(1+e^{-2pu_{K1}}\right)}{e^{2pu_{K1}} - e^{-2pu_{K1}}}\sin pv_1(1-\Delta-2\cos pv_1)$$

The force on conductor one will be directed for x=0 always in y-direction, only the v component is unequal zero.

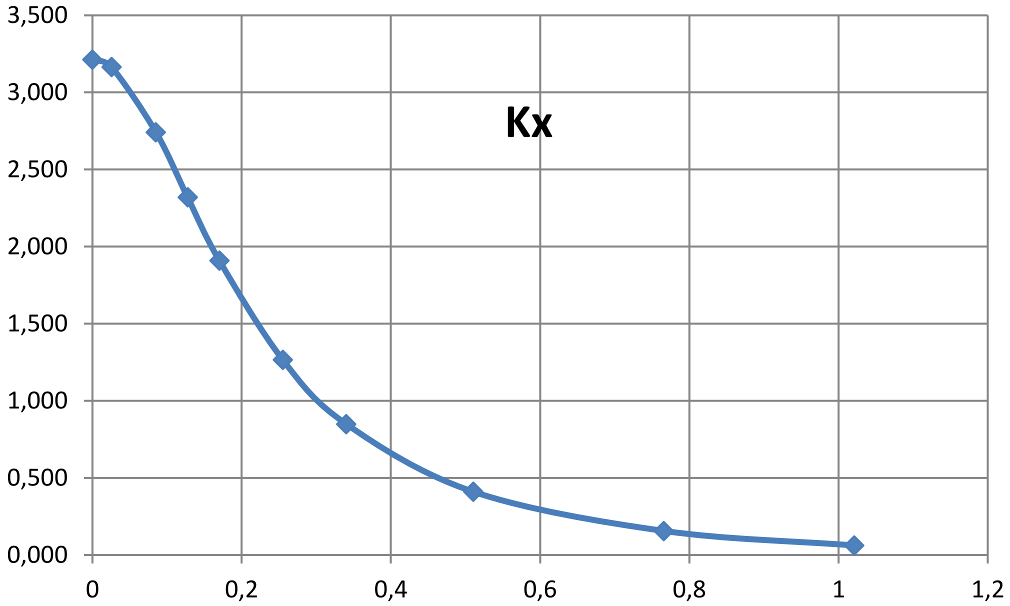
$$\frac{\overrightarrow{dK_1}}{dz} = -\overrightarrow{e_y}\,I_1\frac{\mu_0\,I_1}{2\,\pi}\,\frac{1-\cos v_1}{a}\sum_{p=1}^{\infty}\sin pv_1\left\{(1-\cos pv_1) + \;2\,\frac{(1-\Delta-2\cos pv_1)\left(1+e^{-2pu_{K1}}\right)}{e^{2pu_{K1}} - e^{-2pu_{K1}}}\right\}$$

Example 4: Two conductors between the cylinder

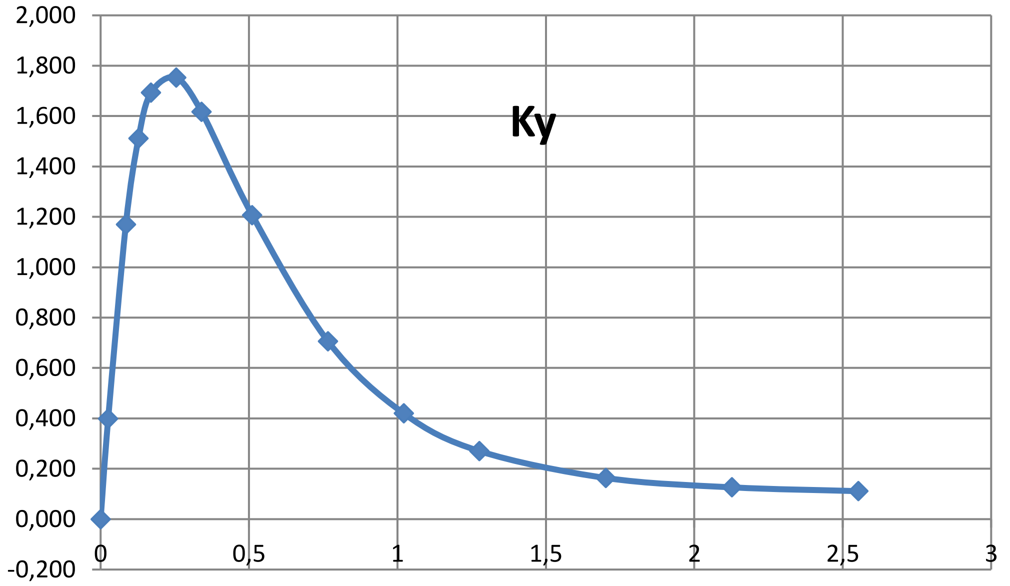
Körper 1		Abstand		**Körper 2**
Radius_1	0,8509	2,626	0,8509	Radius_2
Per_mü1	999		999	Per_mü2
Leiter 1				**Leiter 2**
Strom_1	1		-1	Strom_2
L-ort x1	0,2		-0,2	L-ort x2
L-ort y1	variabel		variabel	L-ort y2



Force in x-direction on conductor 1 between cylinders nearby the y-axis- **all forces normalized to** $\frac{\mu_0}{2\,\pi}$

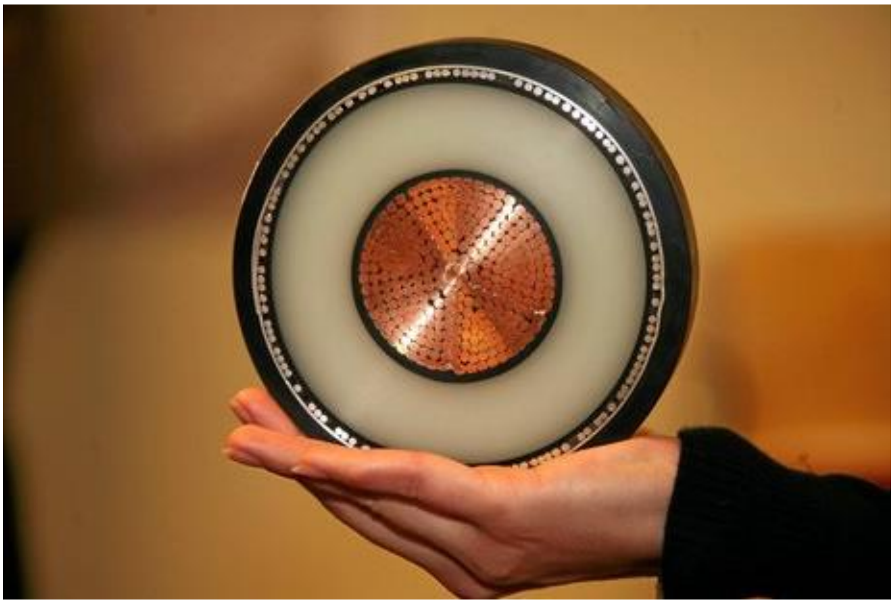


Force in y-direction on conductor 1 between cylinders nearby the y-axis



III.4 Comparison between electrical and weight force

A high voltage power transmission line with power levels of more than 1000 MW (severall kA at one MV) has a size of appoxymateley 200 ... 300 mm with a weight of 5 to 10 kg/ m.



Picture III.1: 380-kV-HGÜ-ground cable, see www.board.bs-netz.com

The force per length unit due to the current is proportional to

$$I \frac{\mu_0 I}{2\pi} = I^2 \frac{4\pi \, 10^{-5}}{2\pi} \frac{Vs}{Am} = 0{,}02 \; 10^{-3} * I^2 \frac{Vs}{Am}$$

For currents in the order of some kA the force on the cable by electrical power transmission is with some resonance effects with a factor of between 0.1 and 10 (see previous examples) will be in the same order or more as the weight force.

For that reason parallel transmission line has to have a certain minimum distance between two lines. This will be more important if there are disturbing elements (or fixing points) with certain permeability in the neighbourhood.

The normal force between two lines with current I_1 and I_2 of length l and distance d is accordance to basic physics

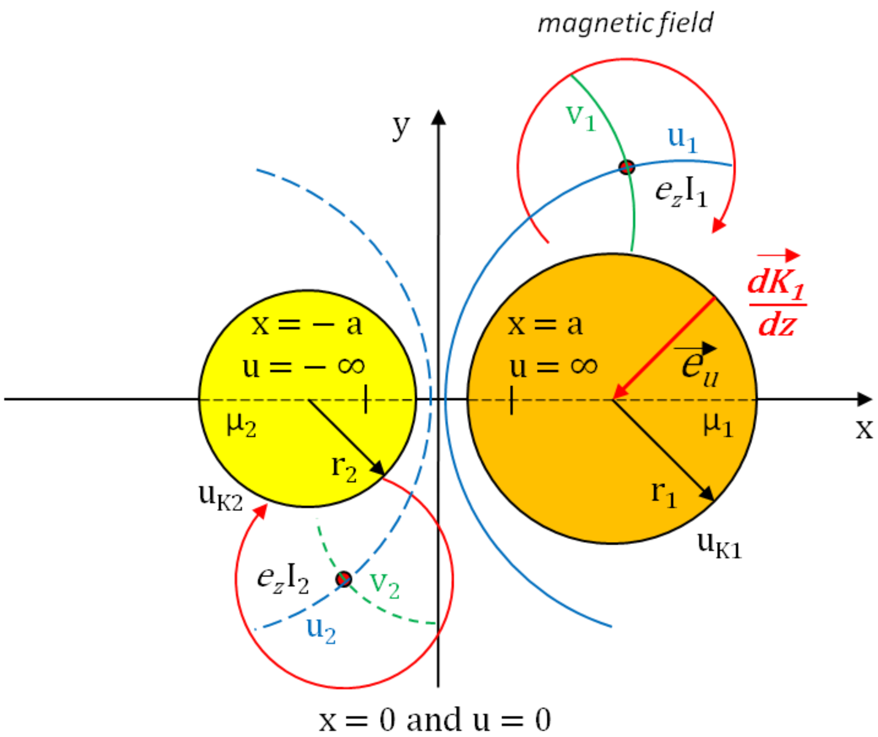
$$F = \frac{\mu_0 \mu_r I_1 I_2}{2\pi} \frac{l}{d}$$

Therefore the force has to be considered during engineering design process of high power transmission lines.

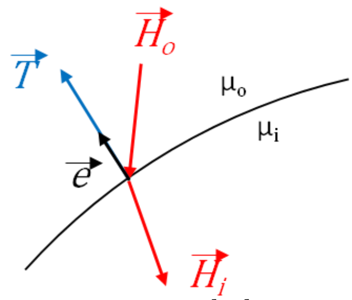
IV. Force on cylinders with given permeability

IV.1 Force on cylinders – general case

The force on a cylinder is a result of the change of permeability at the boundary of the cylinder.



The force on the cylinder will be calculated with the mechanical tension ***T*** on the surface as a result of the magnetic field ***H***.
The force K is the integral of all tensions at the cylinder surface F.



$$\vec{T} = \vec{e}\,\frac{1}{2}\,(\mu_1 - \mu_2) * \left(\vec{H_1} * \vec{H_2}\right)$$

$$\vec{K} = \oiint \vec{T}\ dF$$

In this case we have just a planar problem in u-v coordinate for u = constant = $u_{K1,\,K2}$ with the area element dF in v and z-direction

$$dF = h_v\, dv * h_z\, dz \qquad \text{with metrical factors} \qquad h_v = \frac{a}{\cosh u - \cos v} \qquad h_z = 1$$

Therefore the force on cylinder i=1, 2 per length unit *dz* will be

$$\frac{\overrightarrow{dK_i}}{dz} = \int_{v=-\pi}^{+\pi} \overrightarrow{e_u}\,\frac{1}{2}\left(\mu_0 - \mu_{1,2}\right) * \left(\overrightarrow{H_0} * \overrightarrow{H_{1,2}}\right) h_v dv \qquad \text{(IV.1)}$$

H_0: magnetic field at u=u_{K1} in medium 1 with permeability μ_0 , for example outside cylinder

$H_{1,2}$: magnetic field at u=u_{K1} in medium 2 with permeability $\mu_{1,2}$, inside the cylinder 1 or 2

With the relationship between induction ***B*** = μ ***H*** and magnetic field

Force on cylinder 1:

Taking into account the relationship between magnetic field and induction will transfer the force on the cylinder as follow

$$\frac{\overrightarrow{dK_\iota}}{dz} = \int_{v=-\pi}^{+\pi} \overrightarrow{e_u}\ \frac{1}{2}\ \frac{(\mu_0 - \mu_{1,2})}{\mu_0 * \mu_{1,2}} * \left(\overrightarrow{B_0} * \overrightarrow{B_{1,2}}\right) h_v dv$$

The total magnetic field and the induction outside of the cylinder in accordance to the difference location as in section II.4 and III.1 described will be the sum of the exciting induction of vol. IV. See II.3.4 for the magnetic potential A_{eIV} and "Stör" field $B_{IIStör}$ in vol II (see page 40, equation h)

$$\overrightarrow{B_0} = \overrightarrow{B_{eIV}} + \overrightarrow{B_{IIStör}}$$

The exciting induction B_{eIV} will be with

$$\overrightarrow{B_{eIV}} = rot\ \overrightarrow{A_{eIV}} = \frac{1}{h}\left[\overrightarrow{e_u}\frac{\partial A_{eIV}}{\partial v} - \overrightarrow{e_v}\frac{\partial A_{eIV}}{\partial u}\right]$$

$$\overrightarrow{B_{eIV}} = rot\ \overrightarrow{A_{eIV}} = \frac{1}{h}\sum_{p=1}^{\infty}\left\{\begin{matrix}\overrightarrow{e_u}\frac{\partial}{\partial v}\left[(D_{IVp}\cos pv + E_{IVp}\sin pv)e^{-pu}\right]\ldots \\ -\overrightarrow{e_v}\frac{\partial}{\partial u}\left[(D_{IVp}\cos pv + E_{IVp}\sin pv)e^{-pu}\right]\end{matrix}\right\}$$

$$\overrightarrow{B_{eIV}} = \frac{1}{h}\sum_{p=1}^{\infty} p\left\{\begin{matrix}\overrightarrow{e_u}\left[(-D_{IVp}\sin pv + E_{IVp}\cos pv)e^{-pu}\right]\ldots \\ -\overrightarrow{e_v}(-1)\left[(D_{IVp}\cos pv + E_{IVp}\sin pv)e^{-pu}\right]\end{matrix}\right\}$$

The "Stör" induction $B_{IIStör}$ will be

$$\overrightarrow{B_{IIStör}} = \frac{1}{h}\left\{\begin{matrix}\overrightarrow{e_u}\left[\sum_{p=1}^{\infty} p\left(\begin{matrix}(-C_{pStör}E_{pStör}\sin pv + D_{pStör}E_{pStör}\cos pv)e^{pu} + \\ (-C_{pStör}F_{pStör}\sin pv + D_{pStör}F_{pStör}\cos pv)e^{-pu}\end{matrix}\right)\right] \\ -\overrightarrow{e_v}\left[\sum_{p=1}^{\infty} p\left(\begin{matrix}(C_{pStör}E_{pStör}\cos pv + D_{pStör}E_{pStör}\sin pv)e^{pu} \\ -(C_{pStör}F_{pStör}\cos pv + D_{pStör}F_{pStör}\sin pv)e^{-pu}\end{matrix}\right)\right]\end{matrix}\right\}$$

The outer field/ induction will be therefore the sum of both components

$$\overrightarrow{B_0} = \frac{1}{h}\ p\sum_{p=1}^{\infty}\left\{\begin{matrix}\overrightarrow{e_u}\left[\begin{matrix}(-C_{pStör}E_{pStör}\sin pv + D_{pStör}E_{pStör}\cos pv)e^{pu} + \\ \left[-(C_{pStör}F_{pStör} + D_{IVp})\sin pv + (D_{pStör}F_{pStör} + E_{IVp})\cos pv\right]e^{-pu}\end{matrix}\right] + \\ +\overrightarrow{e_v}\left[\begin{matrix}-(C_{pStör}E_{pStör}\cos pv + D_{pStör}E_{pStör}\sin pv)e^{pu} \\ +\left[(C_{pStör}F_{pStör} + D_{IVp})\cos pv + (D_{pStör}F_{pStör} + E_{IVp})\sin pv\right]e^{-pu}\end{matrix}\right]\end{matrix}\right\}$$

$$\overrightarrow{B_0} = \frac{1}{h}\ p\sum_{p=1}^{\infty}(\overrightarrow{e_u}B_{0u} + \overrightarrow{e_v}B_{0v})$$

The "Stör" field/ induction $B_{IIIStör}$ within the cylinder was already calculated in equation i on page 40. Due to the fact that we have to multiply both fields we are using a new numbering index for the series with q (instead of p)

$$\overrightarrow{B_{IIIStör}} = \frac{1}{h}\left\{\begin{matrix}\overrightarrow{e_u}\left[\sum_{q=1}^{\infty} q\left[-G_{qStör}\sin qv + H_{qStör}\cos qv\right]e^{-qu}\right]\ldots \\ +\overrightarrow{e_v}\sum_{q=1}^{\infty} q\left[G_{qStör}\cos qv + H_{qStör}\sin qv\right]e^{-qu}\end{matrix}\right\}$$

$$\overrightarrow{B_{IIIStör}} = \frac{1}{h}\ q\sum_{q=1}^{\infty}(\overrightarrow{e_u}B_{IIIS\_u} + \overrightarrow{e_v}B_{IIIS\_v})$$

We have to solve the following equation for the force on cylinder 1 (or 2) for $u = u_{K,1,2}$ and $h = h_v$

$$\frac{\overrightarrow{dK_\iota}}{dz} = \int_{v=-\pi}^{+\pi} \overrightarrow{e_u}\ \frac{1}{2}\ \frac{(\mu_0 - \mu_{1,2})}{\mu_0 * \mu_{1,2}}\left[\frac{1}{h_v}p\sum_{p=1}^{\infty}(\overrightarrow{e_u}B_{0u} + \overrightarrow{e_v}B_{0v}) * \frac{1}{h_v}q\sum_{q=1}^{\infty}(\overrightarrow{e_u}B_{IIIS\_u} + \overrightarrow{e_v}B_{IIIS\_v})\right] h_v dv$$

Therefore multiplying both field components will result in the force on cylinder 1

Force on cylinder 1 will be by integration of

$$\frac{\overrightarrow{dK_1}}{dz} = \int_{v=-\pi}^{+\pi} \overrightarrow{e_u}\ \frac{1}{2}\ \frac{(\mu_0-\mu_1)}{\mu_0*\mu_1}\ \frac{\cosh u_{K1}-\cos v}{a}\ \sum_q\sum_p qp\left[\left(\overrightarrow{e_u}\,B_{0u}+\overrightarrow{e_v}\,B_{0v}\right)*\left(\overrightarrow{e_u}\,B_{IIIS\_u}+\overrightarrow{e_v}\,B_{IIIS\_v}\right)\right] dv$$

$$\frac{\overrightarrow{dK_1}}{dz} = \overrightarrow{e_u}\ \frac{1}{2a}\ \frac{(\mu_0-\mu_1)}{\mu_0*\mu_1}\int_{v=-\pi}^{+\pi}(\cosh u_{K1}-\cos v)\ \sum_q\sum_p qp\left[B_{0u}B_{IIIS\_u}+B_{0v}B_{IIIS\_v}\right] dv$$

$$(\dots) \qquad\qquad [\dots]$$

The components are

$$B_{0u} = (-CE_{ps}\sin pv + DE_{ps}\cos pv)e^{pu} + \left[-(CF_{ps}+D_{IVp})\sin pv + (DF_{ps}+E_{IVp})\cos pv\right]e^{-pu}$$

$$B_{0v} = -(CE_{ps}\cos pv + DE_{ps}\sin pv)e^{pu} + \left[+(CF_{ps}+D_{IVp})\cos pv + (DF_{ps}+E_{IVp})\sin pv\right]e^{-pu}$$

$$B_{IIIS\_u} = \left[-G_{qS}\sin qv + H_{qS}\cos qv\right]e^{-qu}$$

$$B_{IIIS\_v} = \left[G_{qS}\cos qv + H_{qS}\sin qv\right]e^{-qu}$$

Multiplying will result in integrals for different combination of sinus, cosines functions for $u = u_{K1}$

$$(\dots)[\dots] = (\cosh u_{K1}-\cos v)\left[[1]+[2]+[3]+[4]\right]$$

$$[1] = \left[\begin{array}{c}(+CE_{ps}\,G_{qS})\sin pv\sin qv\, e^{u_{K1}(p-q)} - DE_{ps}G_{qS}\cos pv\sin qv\ e^{u_{K1}(p-q)}\\ +(CF_{ps}+D_{IVp})G_{qS}\sin pv\ \sin qv\, e^{-u_{K1}(p+q)} - (DF_{ps}+E_{IVp})G_{qS}\cos pv\ \sin qv\, e^{-u_{K1}(p+q)}\end{array}\right]$$

$$[2] = \left[\begin{array}{c}(-CE_{ps}\,H_{qS})\sin pv\cos qv\, e^{u_{K1}(p-q)} + DE_{ps}H_{qS}\cos pv\cos qv\ e^{u_{K1}(p-q)}\\ -(CF_{ps}+D_{IVp})H_{qS}\sin pv\ \cos qv\, e^{-u_{K1}(p+q)} + (DF_{ps}+E_{IVp})H_{qS}\cos pv\ \cos qv\, e^{-u_{K1}(p+q)}\end{array}\right]$$

$$[3] = \left[\begin{array}{c}(-CE_{ps}\,G_{qS})\cos pv\cos qv\, e^{u_{K1}(p-q)} - DE_{ps}G_{qS}\sin pv\cos qv\ e^{u_{K1}(p-q)}\\ +(CF_{ps}+D_{IVp})G_{qS}\cos pv\ \cos qv\, e^{-u_{K1}(p+q)} + (DF_{ps}+E_{IVp})G_{qS}\sin pv\ \cos qv\, e^{-u_{K1}(p+q)}\end{array}\right]$$

$$[4] = \left[\begin{array}{c}(-CE_{ps}\,H_{qS})\cos pv\sin qv\, e^{u_{K1}(p-q)} - DE_{ps}H_{qS}\sin pv\sin qv\ e^{u_{K1}(p-q)}\\ +(CF_{ps}+D_{IVp})H_{qS}\cos pv\ \sin qv\, e^{-u_{K1}(p+q)} + (DF_{ps}+E_{IVp})H_{qS}\sin pv\ \sin qv\, e^{-u_{K1}(p+q)}\end{array}\right]$$

We have the following types of integrals in sinus and cosines as follows:

$Int.1a = \int_{-\pi}^{\pi}\sin pv\sin qv\, dv$ — see Bronstein integral #305

$Int.2a = \int_{-\pi}^{\pi}\sin pv\cos qv\, dv$ — see Bronstein integral #408

$Int.3a = \int_{-\pi}^{\pi}\cos pv\cos qv\, dv$ — see Bronstein integral #346

And with the multiplicand of metric factor (cosh u – cos v) we will get additional six integrals

$$Int.1b = \int_{-\pi}^{\pi}\cos v\sin pv\sin qv\, dv = \int_{-\pi}^{\pi}\cos v\left\{\frac{1}{2}\left[\cos(p-q)v-\cos(p+q)v\right]\right\}dv$$

$$Int.2b = \int_{-\pi}^{\pi}\cos v\sin pv\cos qv\, dv = \int_{-\pi}^{\pi}\cos v\left\{\frac{1}{2}\left[\sin(p-q)v+\sin(p+q)v\right]\right\}dv$$

$$Int.3b = \int_{-\pi}^{\pi}\cos v\cos pv\cos qv\, dv = \int_{-\pi}^{\pi}\cos v\left\{\frac{1}{2}\left[\cos(p-q)v+\cos(p+q)\, v\right]\right\}dv$$

Note: the product of sinus and cosine function will be transferred in a sum.

The solution of integrals will be as follows:

Integral 1a:

$$Int.1a = \int_{-\pi}^{\pi} \sin pv \sin qv\, dv = \begin{cases} \frac{1}{2}\left(v - \frac{1}{2p}\sin 2pv\right) = \frac{1}{2}(\pi+\pi) = \pi \;\; for\ p = q \\ \left[\frac{\sin(p-q)v}{2(p-q)} - \frac{\sin(p+q)v}{2(p+q)}\right] = 0 \qquad for\ p \neq q \end{cases}$$

$$Int.1a = \int_{-\pi}^{\pi} \sin pv \sin qv\, dv = \begin{cases} 0 & for\ p \neq q \\ \pi & for\ p = q \end{cases} \tag{IV.2}$$

Integral 2a:

$$Int.2a = \int_{-\pi}^{\pi} \sin pv \cos qv\, dv = \begin{cases} \frac{1}{2p}\ sin^2\ pv = 0 \qquad for\ p = q \\ -\left[\frac{\cos(p+q)v}{2(p+q)} + \frac{\sin(p-q)v}{2(p-q)}\right] = 0 \;\; for\ p \neq q \end{cases}$$

$$Int.2a = \int_{-\pi}^{\pi} \sin pv \cos qv\, dv = 0 \tag{IV.3}$$

Integral 3a:

$$Int.3a = \int_{-\pi}^{\pi} \cos pv \cos qv\, dv = \begin{cases} \frac{1}{2}\left(v + \frac{1}{2p}\sin 2pv\right) = \frac{1}{2}(\pi+\pi) = \pi \;\; for\ p = q \\ \left[\frac{\sin(p-q)v}{2(p-q)} + \frac{\sin(p+q)v}{2(p+q)}\right] = 0 \qquad for\ p \neq q \end{cases}$$

$$Int.3a = \int_{-\pi}^{\pi} \cos pv \cos qv\, dv = \begin{cases} 0 & for\ p \neq q \\ \pi & for\ p = q \end{cases} \tag{IV.4}$$

The additional three integrals will be

Integral 1b1 and integral 1b2 will be with the results of Int.3a

$$Int.1b \;= \int_{-\pi}^{\pi} \cos\mathrm{v} \sin pv \sin qv\, dv = \int_{-\pi}^{\pi} \frac{1}{2}\cos\mathrm{v}[\cos(p-q)v - \cos(p+q)v]\, dv$$

$$Int.1b1 = \int_{-\pi}^{\pi} \cos\mathrm{v} \left\{\frac{1}{2}[\cos(p-q)v]\right\} dv \;= \begin{cases} \frac{1}{2}\pi & for\ q = p-1 \\ 0 & for\ q \neq p-1 \end{cases} \tag{IV.5a,b}$$

$$Int.1b2 = \int_{-\pi}^{\pi} \cos\mathrm{v} \left\{\frac{1}{2}[-\cos(p+\mathrm{q})v]\right\} dv = \begin{cases} \frac{-1}{2}\pi & for\ q = 1-p \\ 0 & for\ q \neq 1-p \end{cases}$$

with negative p

Integral 2b1 and integral 2b2 will be with the results of Int.2a

$$Int.2b1 = \int_{-\pi}^{\pi} \cos\mathrm{v} \left\{\frac{1}{2}[\sin(p-q)v]\right\} dv = 0$$

$$Int.2b2 = \int_{-\pi}^{\pi} \cos\mathrm{v} \left\{\frac{1}{2}[\sin(p+q)v]\right\} dv = 0$$

$$Int.2b \;= \int_{-\pi}^{\pi} \cos\mathrm{v} \sin pv \cos qv\, dv = \; 0 \tag{IV.6}$$

Integral 3b1 and integral 3b2 will be with the results of Int.3a

$$Int.3b \;= \int_{-\pi}^{\pi} \cos\mathrm{v} \cos pv \cos qv\, dv = \int_{-\pi}^{\pi} \frac{1}{2}\cos\mathrm{v}[\cos(p-q)v + \cos(p+q)\,v]\, dv$$

$$Int.3b1 = \int_{-\pi}^{\pi} \cos\mathrm{v} \left\{\frac{1}{2}[\cos(p-q)v]\right\} dv \;= \begin{cases} \frac{1}{2}\pi & for\ q = p-1 \\ 0 & for\ q \neq p-1 \end{cases} \tag{IV.7 a,b}$$

$$Int.3b2 = \int_{-\pi}^{\pi} \cos\mathrm{v} \left\{\frac{1}{2}[\cos(p+\mathrm{q})v]\right\} dv = \begin{cases} \frac{+1}{2}\pi & for\ q = 1-p \\ 0 & for\ q \neq 1-p \end{cases}$$

with negative p

The force on cylinder one will be with this interim result

$$\frac{d\overrightarrow{K_1}}{dz} = \overrightarrow{e_u}\ \frac{1}{2a}\ \frac{(\mu_0 - \mu_1)}{\mu_0 * \mu_1} \int_{v=-\pi}^{+\pi} \sum_q \sum_p qp\ (\dots)[\dots]\, dv$$

$$(\dots)[\dots] = (\cosh u_{K1} - \cos v)\ [[1] + [2] + [3] + [4]]$$

$$[1] = \begin{bmatrix} CE_{ps}\, G_{qS} \sin pv \sin qv\, e^{u_{K1}(p-q)} - DE_{ps} G_{qS} \cos pv \sin qv\ e^{u_{K1}(p-q)} \\ +(CF_{ps} + D_{IVp}) G_{qS} \sin pv\ \sin qv\, e^{-u_{K1}(p+q)} - (DF_{ps} + E_{IVp}) G_{qS} \cos pv\ \sin qv\, e^{-u_{K1}(p+q)} \end{bmatrix}$$

$$[2] = \begin{bmatrix} -CE_{ps}\, H_{qS} \sin pv \cos qv\, e^{u_{K1}(p-q)} + DE_{ps} H_{qS} \cos pv \cos qv\ e^{u_{K1}(p-q)} \\ -(CF_{ps} + D_{IVp}) H_{qS} \sin pv\ \cos qv\, e^{-u_{K1}(p+q)} + (DF_{ps} + E_{IVp}) H_{qS} \cos pv \cos qv\ e^{-u_{K1}(p+q)} \end{bmatrix}$$

$$[3] = \begin{bmatrix} -CE_{ps}\, G_{qS} \cos pv \cos qv\, e^{u_{K1}(p-q)} - DE_{ps} G_{qS} \sin pv \cos qv\ e^{u_{K1}(p-q)} \\ +(CF_{ps} + D_{IVp}) G_{qS} \cos pv\ \cos qv\, e^{-u_{K1}(p+q)} + (DF_{ps} + E_{IVp}) G_{qS} \sin pv \cos qv\, e^{-u_{K1}(p+q)} \end{bmatrix}$$

$$[4] = \begin{bmatrix} -CE_{ps}\, H_{qS} \cos pv \sin qv\, e^{u_{K1}(p-q)} - DE_{ps} H_{qS} \sin pv \sin qv\ e^{u_{K1}(p-q)} \\ +(CF_{ps} + D_{IVp}) H_{qS} \cos pv\ \sin qv\, e^{-u_{K1}(p+q)} + (DF_{ps} + E_{IVp}) H_{qS} \sin pv \sin qv\ e^{-u_{K1}(p+q)} \end{bmatrix}$$

New sorting

$$[1 \dots 4] = \begin{bmatrix} (CE_{ps}\, G_{qS} - DE_{ps} H_{qS}) \sin pv \sin qv\, e^{u_{K1}(p-q)} \\ +[(CF_{ps} + D_{IVp}) G_{qS} + (DF_{ps} + E_{IVp}) H_{qS}] \sin pv \sin qv\ e^{-u_{K1}(p+q)} \\ + (DE_{ps} H_{qS} - CE_{ps}\, G_{qS}) \cos pv \cos qv\ e^{u_{K1}(p-q)} \\ +[(DF_{ps} + E_{IVp}) H_{qS} + (CF_{ps} + D_{IVp}) G_{qS}] \cos pv \cos qv\ e^{-u_{K1}(p+q)} \\ +(\dots) \cos pv \sin qv\ e^{u_{K1}(p-q)} + (\dots) \cos pv \sin qv\ e^{-u_{K1}(p+q)} \end{bmatrix}$$

We have to solve the following two integral sums

$$Int\_1 = \int_{v=-\pi}^{+\pi} \sum_q \sum_p qp\ \cosh u_{K1} \begin{bmatrix} (CE_{ps}\, G_{qS} - DE_{ps} H_{qS}) \sin pv \sin qv\, e^{u_{K1}(p-q)} \\ +[(CF_{ps} + D_{IVp}) G_{qS} + (DF_{ps} + E_{IVp}) H_{qS}] \sin pv \sin qv\ e^{-u_{K1}(p+q)} \\ + (DE_{ps} H_{qS} - CE_{ps}\, G_{qS}) \cos pv \cos qv\ e^{u_{K1}(p-q)} \\ +[(DF_{ps} + E_{IVp}) H_{qS} + (CF_{ps} + D_{IVp}) G_{qS}] \cos pv \cos qv\ e^{-u_{K1}(p+q)} \\ +(\dots) \cos pv \sin qv\ e^{u_{K1}(p-q)} + (\dots) \cos pv \sin qv\ e^{-u_{K1}(p+q)} \end{bmatrix} dv$$

$$Int\_2 = -\int_{v=-\pi}^{+\pi} \sum_q \sum_p qp\ \cos v \begin{bmatrix} (CE_{ps}\, G_{qS} - DE_{ps} H_{qS}) \sin pv \sin qv\, e^{u_{K1}(p-q)} \\ +[(CF_{ps} + D_{IVp}) G_{qS} + (DF_{ps} + E_{IVp}) H_{qS}] \sin pv \sin qv\ e^{-u_{K1}(p+q)} \\ + (DE_{ps} H_{qS} - CE_{ps}\, G_{qS}) \cos pv \cos qv\ e^{u_{K1}(p-q)} \\ +[(DF_{ps} + E_{IVp}) H_{qS} + (CF_{ps} + D_{IVp}) G_{qS}] \cos pv \cos qv\ e^{-u_{K1}(p+q)} \\ +(\dots) \cos pv \sin qv\ e^{u_{K1}(p-q)} + (\dots) \cos pv \sin qv\ e^{-u_{K1}(p+q)} \end{bmatrix} dv$$

With (IV.2/3/4) the first integral sum will be

$$Int\_1 = \sum_{q=p} \sum_p qp\ \cosh u_{K1}\, \pi \begin{bmatrix} (CE_{ps}\, G_{qS} - DE_{ps} H_{qS}) \\ +[(CF_{ps} + D_{IVp}) G_{qS} + (DF_{ps} + E_{IVp}) H_{qS}]\ e^{-2pu_{K1}} \\ + (DE_{ps} H_{qS} - CE_{ps}\, G_{qS}) \\ +[(DF_{ps} + E_{IVp}) H_{qS} + (CF_{ps} + D_{IVp}) G_{qS}]\ e^{-2pu_{K1}} \\ + 0 + 0 \end{bmatrix}$$

$$Int\_1 = 2\pi \cosh u_{K1} \sum_{p=1}^{\infty} p^2 [(CF_{ps} + D_{IVp}) G_{qS} + (DF_{ps} + E_{IVp}) H_{qS}]\ e^{-2pu_{K1}}$$

The second integral will have cos v times sin x sin y and cos v times cos x cos y

$$Int\_2 = -\int_{v=-\pi}^{+\pi} \sum_q \sum_p qp \cos v \begin{bmatrix} (CE_{ps}\,G_{qS} - DE_{ps}H_{qS})\cos(p-q)\,v\,e^{u_{K1}(p-q)} \\ -(CE_{ps}\,G_{qS} - DE_{ps}H_{qS})\cos(p+q)\,v\,e^{u_{K1}(p-q)} \\ +[(CF_{ps}+D_{IVp})G_{qS} + (DF_{ps}+E_{IVp})H_{qS}]\cos(p-q)\;e^{-u_{K1}(p+q)} \\ -[(CF_{ps}+D_{IVp})G_{qS} + (DF_{ps}+E_{IVp})H_{qS}]\cos(p+q)\;e^{-u_{K1}(p+q)} \\ +(DE_{ps}H_{qS} - CE_{ps}\,G_{qS})\cos(p-q)\;e^{u_{K1}(p-q)} \\ +(DE_{ps}H_{qS} - CE_{ps}\,G_{qS})\cos(p+q)\;e^{u_{K1}(p-q)} \\ +[(DF_{ps}+E_{IVp})H_{qS} + (CF_{ps}+D_{IVp})G_{qS}]\cos(p-q)\;e^{-u_{K1}(p+q)} \\ +[(DF_{ps}+E_{IVp})H_{qS} + (CF_{ps}+D_{IVp})G_{qS}]\cos(p+q)\;e^{-u_{K1}(p+q)} \end{bmatrix} dv$$

This will result in two series with $q = p - 1$ for the difference of $p-q$ and $q = 1 - p$ for the sum of $p+q$

$$Int\_2a = -\frac{\pi}{2}\sum_{q=p-1}\sum_p qp \begin{bmatrix} (CE_{ps}\,G_{qS} - DE_{ps}H_{qS})e^{u_{K1}} \\ +[(CF_{ps}+D_{IVp})G_{qS} + (DF_{ps}+E_{IVp})H_{qS}]e^{-u_{K1}(2p-1)} \\ +(DE_{ps}H_{qS} - CE_{ps}\,G_{qS})e^{u_{K1}} \\ +[(DF_{ps}+E_{IVp})H_{qS} + (CF_{ps}+D_{IVp})G_{qS}]e^{-u_{K1}(2p-1)} \end{bmatrix}$$

$$Int\_2a = -\pi \sum_{p=1}^{\infty} p(p-1)[(CF_{ps}+D_{IVp})G_{qS} + (DF_{ps}+E_{IVp})H_{qS}]e^{-u_{K1}(2p-1)}$$

And the second integral sum will be

$$Int\_2b = -\frac{\pi}{2}\sum_{q=1-p}\sum_p qp \begin{bmatrix} -(CE_{ps}\,G_{qS} - DE_{ps}H_{qS})e^{u_{K1}(2p-1)} \\ -[(CF_{ps}+D_{IVp})G_{qS} + (DF_{ps}+E_{IVp})H_{qS}]e^{-u_{K1}} \\ +(DE_{ps}H_{qS} - CE_{ps}\,G_{qS})e^{u_{K1}(2p-1)} \\ +[(DF_{ps}+E_{IVp})H_{qS} + (CF_{ps}+D_{IVp})G_{qS}]e^{-u_{K1}} \end{bmatrix}$$

$$Int\_2b = -\pi \sum_{p=1}^{\infty} p(p-1)(CE_{ps}\,G_{qS} - DE_{ps}H_{qS})e^{u_{K1}(2p-1)}$$

The force on cylinder one will be with these three integral sums at the end

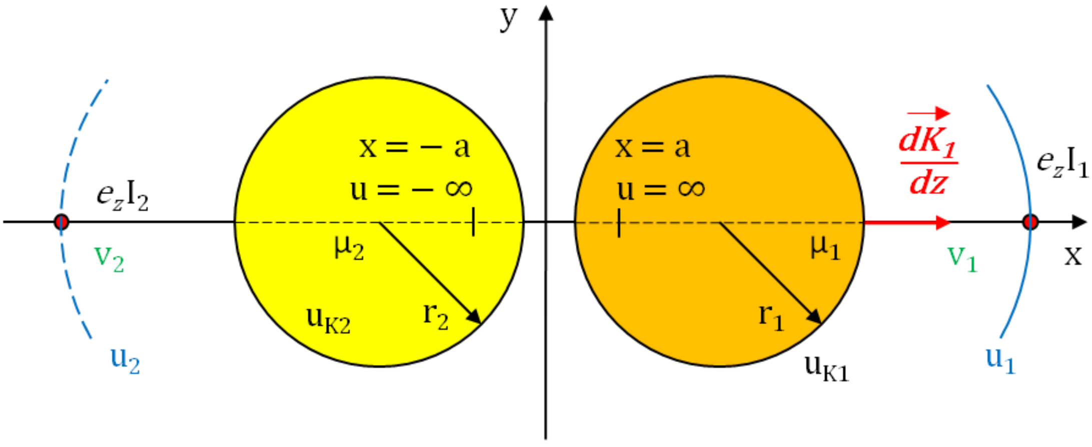
$$\frac{d\overrightarrow{K_1}}{dz} = \overrightarrow{e_u}\,\frac{\pi}{2a}\,\frac{(\mu_0 - \mu_1)}{\mu_0 * \mu_1}\begin{bmatrix} 2\cosh u_{K1}\sum_{p=1}^{\infty} p^2[(CF_{ps}+D_{IVp})G_{qS} + (DF_{ps}+E_{IVp})H_{qS}]\;e^{-2pu_{K1}} \\ -\sum_{p=1}^{\infty} p(p-1)[(CF_{ps}+D_{IVp})G_{qS} + (DF_{ps}+E_{IVp})H_{qS}]e^{-u_{K1}(2p-1)} \\ -\sum_{p=1}^{\infty} p(p-1)(CE_{ps}\,G_{qS} - DE_{ps}H_{qS})e^{u_{K1}(2p-1)} \end{bmatrix} \qquad (IV.8)$$

The constants of the exciting magnetic field of two currents in front of the cylinder and the disturbing "Stör" magnetic field are already known, see section II.1 until II.4

$CF_{pStör}$	II.4.6	$f(D_{Ip}, D_{IVp})$	II.3.5a,b
D_{IVp}	II.3.5b	basic functiona s of	$f(A_p, B_p, A^*_p, B^*_p)$
G_{qs}	II.4.9	$f(D_{IVp}, CE_{pStör}, CF_{pStör})$	
$DF_{pStör}$	II.4.10	$f(E_{Ip}, E_{IVp})$	II.3.6a,b
E_{IVp}	II.3.6b	basic function as of	$f(C_p, C^*_p)$
H_{qs}	II.4.13	$f(E_{IVp}, DE_{pStör}, DF_{pStör})$	
$CE_{pStör}$	II.4.7	$f(D_{IVp}, CF_{pStör})$	
$DE_{pStör}$	II.4.11	$f(E_{IVp}, DF_{pStör})$	

All are based on the exciting basic coefficients as of equation II.1.5a,b,c and II.2.1a,b,c

IV.2 Example: Force on cylinders



In this example we would like to calculate the force on cylinder if both conductors with $+/-I$ are located on the x-axis for $y = 0$ in front of cylinder with high permeability.

We will use the same dimensions as in section III.2 for the force on conductors as follows:

D = 2,626 (metre, yard,)	r = 0,8509 (metre, yard,)
The geometric constant a will be	a = 1 (metre, yard,)
The cylinder contour will be at	$u_{K1,2} = \pm 1$
The conductors are located at $y_{L1,2} = 0$	$v_{L1,2} = \pi$ for x<a and 0 for x>a

For the case of conductors on the x-axis the force vector will be with I.6.3/4

$$V_x = \frac{(1-\cosh u \cos v)\, V_u - \sinh u \sin v\, V_v}{\cosh u - \cos v} = V_u \frac{(1-\cosh u \cos v)}{\cosh u - \cos v} = \begin{cases} +V_u & for\ x < a \\ -V_u & for\ x > a \end{cases} \quad \text{(IV.9)}$$

$$V_y = \frac{\sinh u \sin v\ V_u + (1-\cosh u \cos v)\, V_v}{\cosh u - \cos v} = V_u \frac{\sinh u \sin v}{\cosh u - \cos v} = 0$$

We will have a force in x direction, depending on the location of conductors either in $\pm$ x-axis.

a.) Force on cylinder 1 between the cylinder $\quad 0 < x < D/2 - r$

The coefficients of the different equations are in accordance with below mentioned equations.

I_1	$I_2 = -I_1$	u_1	$u_2 = -u_1$	$v_{1,2} = \pi$
u_{K1}	$u_{K2} = -u_{K1}$	contour coordinates of cylinder		$\mu_{1,2} = \infty$

II.1.5 Coefficients of exciting field with current I_1

$$A_p = -\frac{\mu_0 I_1}{2\pi}\frac{1}{p} \qquad B_p = \frac{\mu_0 I_1}{2\pi}\frac{(-1)^p}{p e^{p u_1}} = -A_p \frac{(-1)^p}{e^{p u_1}} \qquad C_p = \frac{\mu_0 I_1}{2\pi}\frac{\sin p v_1}{p e^{p u_1}} = 0$$

II.2.1 Coefficients of exciting field with current I_2

$$A_p^* = -\frac{\mu_0 I_2}{2\pi}\frac{1}{p} \qquad B_p^* = \frac{\mu_0 I_2}{2\pi}\frac{(-1)^p}{p e^{-p(-u_1)}} = -A_p^* \frac{(-1)^p}{e^{p u_1}} \qquad C_p^* = \frac{\mu_0 I_2}{2\pi}\frac{\sin p v_2}{p e^{-p u_2}} = 0$$

$$A_p^* = -\frac{\mu_0 (-I_1)}{2\pi}\frac{1}{p} = -A_p \qquad B_p^* = +A_p \frac{(-1)^p}{e^{p u_1}}$$

II.3.5 Coefficients of exciting field with two conductors with current $I_{1,2}$

$$D_{Ip} = A_p + B_p + A_p^* + B_p^* e^{-2pu_2} = A_p \left[-\frac{(-1)^p}{e^{pu_1}} + \frac{(-1)^p}{e^{pu_1}} e^{2pu_1}\right]$$

$$D_{Ip} = A_p \left[(-e^{-pu_1} + e^{pu_1})(-1)^p\right] = A_p \left(e^{pu_1} - e^{-pu_1}\right)(-1)^p \quad >>$$

$$D_{IVp} = A_p + B_p e^{2pu_1} + A_p^* + B_p^* = A_p \left[-\frac{(-1)^p}{e^{pu_1}} e^{2pu_1} + \frac{(-1)^p}{e^{pu_1}}\right]$$

$$D_{IVp} = A_p \left[(-e^{pu_1} + e^{pu_1})(-1)^p\right] = A_p \left(e^{pu_1} - e^{-pu_1}\right)(-1)^p = D_{Ip} \quad >>$$

II.3.6 Coefficients of exciting field with two conductors with current $I_{1,2}$

$$E_{Ip} = C_p + C_p^* e^{-2pu_2} = 0 \qquad E_{IVp} = C_p e^{2pu_1} + C_p^* = 0$$

II.4.6 Coefficients of "Stör" field with two conductors with current $I_{1,2}$

$$C_{pStör} F_{pStör} = \frac{D_{Ip} + D_{IVp} e^{-2pu_{K1}}}{e^{2pu_{K1}} - e^{-2pu_{K1}}} = \frac{D_{Ip} + D_{Ip} e^{-2pu_{K1}}}{e^{2pu_{K1}} - e^{-2pu_{K1}}} = D_{Ip} \frac{1 + e^{-2pu_{K1}}}{1 - e^{-4pu_{K1}}} e^{-2pu_{K1}} \quad >>$$

II.4.7 Coefficients of "Stör" field with two conductors with current $I_{1,2}$

$$C_{pStör} E_{pStör} = \left(D_{IVp} + C_{pStör} F_{pStör}\right) e^{-2pu_{K1}} = \left(D_{Ip} + D_{Ip} \frac{1 + e^{-2pu_{K1}}}{e^{2pu_{K1}} - e^{-2pu_{K1}}}\right) e^{-2pu_{K1}}$$

$$C_{pStör} E_{pStör} = D_{Ip} \left[\frac{\left(e^{2pu_{K1}} - e^{-2pu_{K1}}\right)}{e^{2pu_{K1}} - e^{-2pu_{K1}}} + \frac{1 + e^{-2pu_{K1}}}{e^{2pu_{K1}} - e^{-2pu_{K1}}}\right] e^{-2pu_{K1}}$$

$$C_{pStör} E_{pStör} = D_{Ip} \left[\frac{e^{2pu_{K1}}}{e^{2pu_{K1}} - e^{-2pu_{K1}}} + \frac{1}{e^{2pu_{K1}} - e^{-2pu_{K1}}}\right] e^{-2pu_{K1}} = CF_{pS} \quad >>$$

II.4.9 Coefficients of "Stör" field with two conductors with current $I_{1,2}$

$$G_{pStör} = D_{IVp} + C_{pStör} E_{pStör} e^{2pu_{K1}} + C_{pStör} F_{pStör} = D_{Ip} + CF_{pStör}\left(e^{2pu_{K1}} + 1\right)$$

$$G_{pStör} = D_{Ip} + D_{Ip} \frac{1 + e^{-2pu_{K1}}}{e^{2pu_{K1}} - e^{-2pu_{K1}}}\left(e^{2pu_{K1}} + 1\right) = D_{Ip} + D_{Ip} \frac{e^{2pu_{K1}} + 1 + 1 + e^{-2pu_{K1}}}{e^{2pu_{K1}} - e^{-2pu_{K1}}} =$$

$$G_{pStör} = D_{Ip} \frac{e^{2pu_{K1}} - e^{-2pu_{K1}} + e^{2pu_{K1}} + 2 + e^{-2pu_{K1}}}{e^{2pu_{K1}} - e^{-2pu_{K1}}} = \frac{2\, D_{Ip}\left(1 + e^{2pu_{K1}}\right)}{e^{2pu_{K1}} - e^{-2pu_{K1}}} = 2\, D_{Ip} \frac{1 + e^{-2pu_{K1}}}{1 - e^{-4pu_{K1}}}$$

II.4.10 Coefficients of "Stör" field with two conductors with current $I_{1,2}$

$$D_{pStör} F_{pStör} = \frac{E_{Ip} e^{p(u_{K1} + u_{K2})} + \left(\frac{\mu_1 - \mu_0}{\mu_1 + \mu_0}\right) E_{IVp} e^{-p(u_{K1} - u_{K2})}}{\left(\frac{\mu_2 + \mu_0}{\mu_2 - \mu_0}\right) e^{p(u_{K1} - u_{K2})} - \left(\frac{\mu_1 - \mu_0}{\mu_1 + \mu_0}\right) e^{-p(u_{K1} - u_{K2})}} = 0$$

because E = 0

II.4.11 Coefficients of "Stör" field with two conductors with current $I_{1,2}$

$$D_{pStör} E_{pStör} = \left(E_{IVp} + D_{pStör} F_{pStör}\right) \left(\frac{\mu_1 - \mu_0}{\mu_1 + \mu_0}\right) e^{-2pu_{K1}} = 0$$

because E and DF = 0

II.4.13 Coefficients of "Stör" field with two conductors with current $I_{1,2}$

$$H_{pStör} = E_{IVp} + D_{pStör} E_{pStör} e^{2pu_{K1}} + D_{pStör} F_{pStör} = 0$$

because E, DE and DF = 0

With these values the equation for the force on cylinder one will be reduced to

$$\frac{\overrightarrow{dK_1}}{dz} = \overrightarrow{e_u} \frac{\pi}{2a} \frac{(\mu_0 - \mu_1)}{\mu_0 * \mu_1} \begin{bmatrix} 2 \cosh u_{K1} \sum_{p=1}^{\infty} p^2 \left[\left(CF_{pS} + D_{IVp}\right) G_{qS}\right] e^{-2pu_{K1}} \\ - \sum_{p=1}^{\infty} p(p-1) \left[\left(CF_{pS} + D_{IVp}\right) G_{qS}\right] e^{-u_{K1}(2p-1)} \\ - \sum_{p=1}^{\infty} p(p-1) \left(CE_{pS}\, G_{qS}\right) e^{u_{K1}(2p-1)} \end{bmatrix} = \cdots \begin{bmatrix} +S1 \\ -S2 \\ -S3 \end{bmatrix}$$

The coefficients are

$$C_{pStör}F_{pStör} = \frac{D_{Ip}e^{p(u_{K1}+u_{K2})}+\left(\frac{\mu_1-\mu_0}{\mu_1+\mu_0}\right)D_{IVp}e^{-p(u_{K1}-u_{K2})}}{\left(\frac{\mu_2+\mu_0}{\mu_2-\mu_0}\right)e^{p(u_{K1}-u_{K2})}-\left(\frac{\mu_1-\mu_0}{\mu_1+\mu_0}\right)e^{-p(u_{K1}-u_{K2})}}$$

$$C_{pStör}E_{pStör} = \left(D_{IVp} + C_{pStör}F_{pStör}\right)\left(\frac{\mu_1-\mu_0}{\mu_1+\mu_0}\right)e^{-2pu_{K1}}$$

$$G_{pStör} = D_{IVp} + C_{pStör}E_{pStör}e^{2pu_{K1}} + C_{pStör}F_{pStör}$$

$$D_{Ip} = A_p + B_p + A_p^* + B_p^* e^{-2pu_2}$$

$$D_{IVp} = A_p + B_p e^{2pu_1} + A_p^* + B_p^*$$

$$G_{ps} = D_{IVp} + CE_{ps}e^{2pu_{K1}} + CF_{ps} = D_{Ip} + CF_{ps}\left(1+e^{2pu_{K1}}\right)$$

$$= D_{Ip} + D_{Ip}\frac{1+e^{-2pu_{K1}}}{1-e^{-4pu_{K1}}}\,e^{-2pu_{K1}}\left(1+e^{2pu_{K1}}\right) = D_{Ip}\left[\frac{1-e^{-4pu_{K1}}}{1-e^{-4pu_{K1}}} + \frac{\left(1+e^{-2pu_{K1}}\right)\left(1+e^{-2pu_{K1}}\right)}{1-e^{-4pu_{K1}}}\right]$$

$$G_{ps} = D_{Ip}\left[\frac{1-e^{-4pu_{K1}}+1+2e^{-2pu_{K1}}+e^{-4pu_{K1}}}{1-e^{-4pu_{K1}}}\right] = 2\,D_{Ip}\frac{1+e^{-2pu_{K1}}}{1-e^{-4pu_{K1}}}$$

$$CF_{ps} + D_{IVp} = D_{Ip}\frac{1+e^{-2pu_{K1}}}{1-e^{-4pu_{K1}}}\,e^{-2pu_{K1}} + D_{Ip} = D_{Ip}\frac{\left(e^{-2pu_{K1}}+e^{-4pu_{K1}}\right)+1-e^{-4pu_{K1}}}{1-e^{-4pu_{K1}}} = D_{Ip}\frac{1+e^{-2pu_{K1}}}{1-e^{-4pu_{K1}}}$$

For q = p we will have

$$\left[\left(CF_{ps}+D_{IVp}\right)G_{qS}\right]e^{-2pu_{K1}} = D_{Ip}\frac{1+e^{-2pu_{K1}}}{1-e^{-4pu_{K1}}}\,2\,D_{Ip}\frac{1+e^{-2pu_{K1}}}{1-e^{-4pu_{K1}}}e^{-2pu_{K1}}$$

$$\left[\left(CF_{ps}+D_{IVp}\right)G_{qS}\right]e^{-2pu_{K1}} = 2\left(D_{Ip}\frac{1+e^{-2pu_{K1}}}{1-e^{-4pu_{K1}}}\right)^2 e^{-2pu_{K1}}$$

For q = p − 1 we will have

$$\left[\left(CF_{ps}+D_{IVp}\right)G_{qS}\right]e^{-u_{K1}(2p-1)} = D_{Ip}\frac{1+e^{-2pu_{K1}}}{1-e^{-4pu_{K1}}}\,2\,D_{Iq}\frac{1+e^{-2qu_{K1}}}{1-e^{-4qu_{K1}}}\,e^{-u_{K1}(2p-1)}$$

$$= D_{Ip}\frac{1+e^{-2pu_{K1}}}{1-e^{-4pu_{K1}}}\,2\,D_{Iq}\frac{1+e^{-2(p-1)u_{K1}}}{1-e^{-4(p-1)u_{K1}}}e^{-u_{K1}(2p-1)} = 2D_{Ip}D_{Iq}\frac{\left(1+e^{-2pu_{K1}}\right)}{1-e^{-4pu_{K1}}}\frac{\left(e^{-u_{K1}(2p-1)}+e^{-(2p-2+2p-1)u_{K1}}\right)}{1-e^{-4(p-1)u_{K1}}}$$

$$\left[\left(CF_{ps}+D_{IVp}\right)G_{qS}\right]e^{-u_{K1}(2p-1)} = 2D_{Ip}D_{Iq}\frac{\left(1+e^{-2pu_{K1}}\right)\left(1+e^{-2(p-1)u_{K1}}\right)}{\left(1-e^{-4pu_{K1}}\right)\left(1-e^{-4(p-1)u_{K1}}\right)}e^{-u_{K1}(2p-1)} \quad >>$$

For q = 1 − p we will have

$$\left(CE_{ps}\,G_{qS}\right)e^{u_{K1}(2p-1)} = D_{Ip}\frac{1+e^{-2pu_{K1}}}{1-e^{-4pu_{K1}}}\,e^{-2pu_{K1}}2\,D_{Iq}\frac{1+e^{-2qu_{K1}}}{1-e^{-4qu_{K1}}}e^{u_{K1}(2p-1)}$$

$$= D_{Ip}\frac{1+e^{-2pu_{K1}}}{1-e^{-4pu_{K1}}}\,e^{-2pu_{K1}}2\,D_{Iq}\frac{1+e^{-2(1-p)u_{K1}}}{1-e^{-4(1-p)u_{K1}}}e^{u_{K1}(2p-1)}$$

$$= D_{Ip}\frac{1+e^{-2pu_{K1}}}{1-e^{-4pu_{K1}}}\,e^{-2pu_{K1}}2\,D_{Iq}\frac{1+e^{+2(p-1)u_{K1}}}{1-e^{+4(p-1)u_{K1}}}e^{u_{K1}(2p-1)}$$

$$= D_{Ip}\frac{1+e^{-2pu_{K1}}}{1-e^{-4pu_{K1}}}\,e^{-2pu_{K1}}2\,D_{Iq}\frac{e^{2(p-1)u_{K1}}}{e^{4(p-1)u_{K1}}}\frac{e^{-2(p-1)u_{K1}}+1}{e^{-4(p-1)u_{K1}}-1}e^{u_{K1}(2p-1)}$$

$$= -D_{Ip}\frac{1+e^{-2pu_{K1}}}{1-e^{-4pu_{K1}}}\,e^{-2pu_{K1}}2\,D_{Iq}\frac{1+e^{-2(p-1)u_{K1}}}{1-e^{-4(p-1)u_{K1}}}e^{u_{K1}(2p-1)}e^{-2(p-1)u_{K1}}$$

$$= -\,2D_{Ip}D_{Iq}\frac{1+e^{-2pu_{K1}}}{1-e^{-4pu_{K1}}}\frac{1+e^{-2(p-1)u_{K1}}}{1-e^{-4(p-1)u_{K1}}}e^{u_{K1}(2p-1-2p+2-2p)}$$

$$\left(CE_{ps}\,G_{qS}\right)e^{u_{K1}(2p-1)} = -\,2D_{Ip}D_{Iq}\frac{1+e^{-2pu_{K1}}}{1-e^{-4pu_{K1}}}\frac{1+e^{-2(p-1)u_{K1}}}{1-e^{-4(p-1)u_{K1}}}e^{-u_{K1}(2p-1)} \quad >>$$

Therefore the equation become finally with sum S2 = − sum S3

$$\frac{\overrightarrow{dK_1}}{dz} = \overrightarrow{e_u}\,\frac{\pi}{2a}\,\frac{(\mu_0-\mu_1)}{\mu_0 * \mu_1}\begin{bmatrix} 2\cosh u_{K1}\sum_{p=1}^{\infty} p^2\left[\left(CF_{pS}+D_{IVp}\right)G_{qS}\right]e^{-2pu_{K1}} \\ -\sum_{p=1}^{\infty} p(p-1)\left[\left(CF_{pS}+D_{IVp}\right)G_{qS}\right]e^{-u_{K1}(2p-1)} \\ -\sum_{p=1}^{\infty} p(p-1)\left(CE_{pS}\,G_{qS}\right)e^{u_{K1}(2p-1)} \end{bmatrix} = \cdots \begin{bmatrix} +S1 \\ -S2 \\ -S3 \end{bmatrix}$$

$$\frac{\overrightarrow{dK_1}}{dz} = \overrightarrow{e_u}\,\frac{\pi}{2a}\,\frac{(\mu_0-\mu_1)}{\mu_0 * \mu_1}\,2\cosh u_{K1}\sum_{p=1}^{\infty} p^2 2\left(D_{Ip}\,\frac{1+e^{-2pu_{K1}}}{1-e^{-4pu_{K1}}}\right)^2 e^{-2pu_{K1}}$$

$$\frac{\overrightarrow{dK_1}}{dz} = \overrightarrow{e_u}\,\frac{\pi}{2a}\,\frac{(\mu_0-\mu_1)}{\mu_0 * \mu_1}\,2\cosh u_{K1}\sum_{p=1}^{\infty} p^2 2\left(A_p\,\left(e^{pu_1}-e^{-pu_1}\right)\frac{1+e^{-2pu_{K1}}}{1-e^{-4pu_{K1}}}\right)^2 e^{-2pu_{K1}}$$

$$\boxed{\frac{\overrightarrow{dK_1}}{dz} = -\,\overrightarrow{e_u}\,\frac{2\pi}{a}\,\frac{(\mu_1-\mu_0)}{\mu_1 * \mu_0}\cosh u_{K1}\left(\frac{\mu_0\,I_1}{2\,\pi}\right)^2\sum_{p=1}^{\infty}\left[\left(e^{pu_1}-e^{-pu_1}\right)\frac{1+e^{-2pu_{K1}}}{1-e^{-4pu_{K1}}}\right]^2 e^{-2pu_{K1}}} \quad \text{(IV.10)}$$

b.) Force on cylinder 1 outside the cylinder $D/2 + r < x < \infty$

The coefficients of the different equations are in accordance with below mentioned equations.

I_1	$I_2 = -I_1$	u_1	$u_2 = -u_1$	$v_{1,2} = 0$
u_{K1}	$u_{K2} = -u_{K1}$	contour coordinates of cylinder		$\mu_{1,2} = \infty$

II.1.5 Coefficients of exciting field with current I_1

$$A_p = -\frac{\mu_0\,I_1}{2\,\pi}\,\frac{1}{p} \qquad B_p = \frac{\mu_0 I_1}{2\pi}\,\frac{1}{pe^{pu_1}} = -A_p\,\frac{1}{e^{pu_1}} \qquad C_p = \frac{\mu_0 I_1}{2\pi}\,\frac{\sin pv_1}{pe^{pu_1}} = 0$$

All other coefficients will be the same as in case for force on cylinder between the cylinders.

Due to the fact that in this case the coefficient A_p as part of D_{Ip} will be squared in the above equation the force will be described by the same equation.

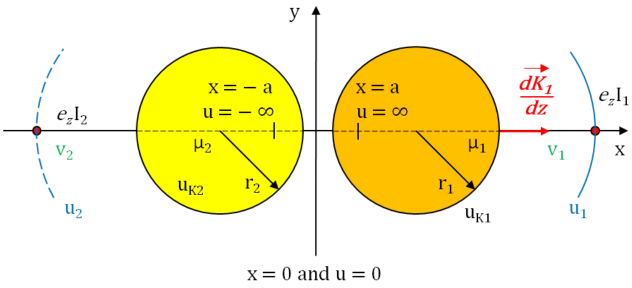
The difference between both cases is the calculation of the location of the conductor on x-axis with equation I.2.6 for u as function of x and vice versa with I.1.2

With the metric factor h and equation IV.9 for the components in x and y direction we get finally for the force on cylinder if the conductor is between the cylinder or outside (x > a) only a force in x-direction:

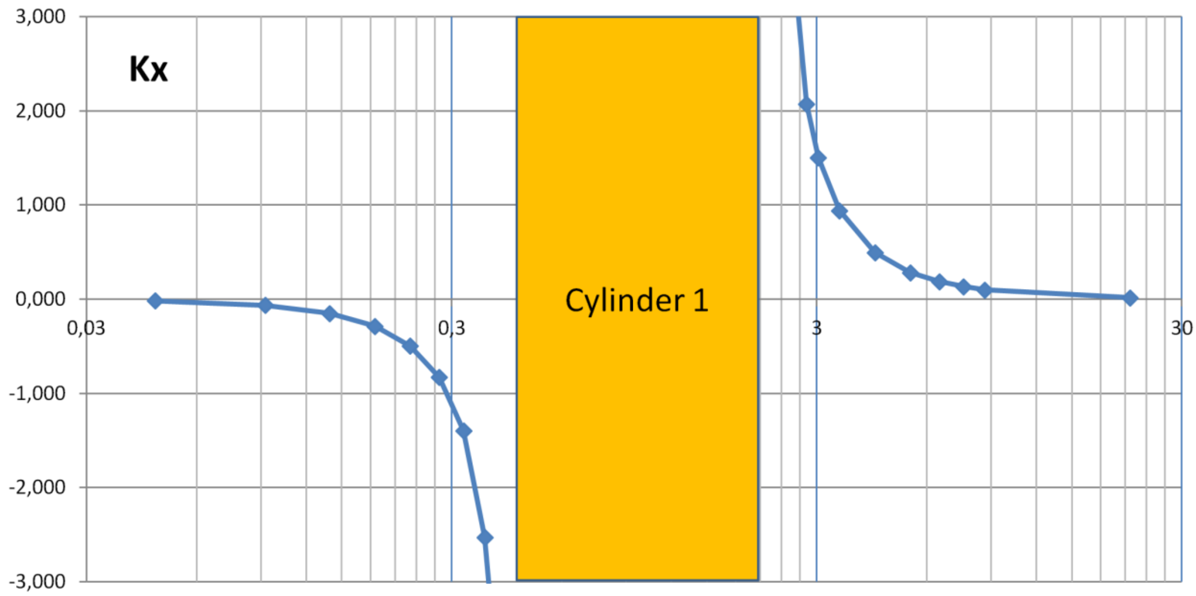
$$\boxed{\frac{\overrightarrow{dK_{1x}}}{dz} = \begin{cases} +\dfrac{\overrightarrow{dK_{1u}}}{dz} & for\ x < a \\ -\dfrac{\overrightarrow{dK_{1u}}}{dz} & for\ x > a \end{cases}} \quad \text{(IV.11)}$$

Example 5: force on the cylinder

Körper 1		Abstand		Körper 2
Radius_1	0,8509	2,626	0,8509	Radius_2
Per_mü1	999		999	Per_mü2
Leiter 1				**Leiter 2**
Strom_1	1		-1	Strom_2
L-ort x1	variabel		- - -	L-ort x2
L-ort y1	0		0	L-ort y2



Force on cylinder 1 with conductor between cylinder and outside in positive x-plane



Note: The force is normalised to the multiplicand $\frac{2\pi}{a}\frac{(\mu_1-\mu_0)}{\mu_1 * \mu_0}\left(\frac{\mu_0\, I_1}{2\,\pi}\right)^2$ as of equation IV.10

V. References

1. Theoretische Elektrotechnik
   Prof. L. Hanakam, script at TU Berlin; 1976

2. Field Theory Handbook
   P. Moon and D. E. Spencer; Springer-Verlag Berlin Heidelberg New York.; 1988

3. Taschenbuch der Mathematik
   I.N. Bronstein und K.A. Semendjaew;
   available in different versions by different Verlagen/ press companies

   Verlag B.G. Teubner (ehemals Verlag Harri Deutsch); ISBN 3-519-20012-0
   also available in English by Oxford University Press ISBN 978-0198507635

   Springer Verlag (in English) ISBN 978-3662462201 6th Edition

4. Excel program files for

Bipolar Cylindrical Coordinates:	Bipolar_Coordinates_v1.xlsx
Force on conductor in x-direction:	Kraft_Leiter_x.xlsx
Force on conductor in y-direction:	Kraft_Leiter_y.xlsx
Force on cylinder in x-direction:	Kraft_Körper_x.xlsx

5. Drawings and pictures Bipolar_Cylindrical_Graphiken.pptx

Attachments – numerical calculation

Example 1: Force on conductor without cylinder — as of page 49

File: Kraft_Leiter_x.xlsx

Körper 1		Abstand		**Körper 2**	
Radius_1	0,8509	2,626	0,8509	Radius_2	
Per_mü1	1,001		1,001	Per_mü2	
Leiter 1				**Leiter 2**	
Strom_1	1		-1	Strom_2	
L-ort x1	variabel		- - -	L-ort x2	
L-ort y1	0		0	L-ort y2	spiegelbildlich zu Leiter 1

	<<< zwischen den Zylindern >>>									<<< Leiter in positiver x-Halbebene >>>								
	0,15	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,85	1,25	1,5	1,75	2	2,5	3	4	6	10
x	0,069	0,092	0,139	0,185	0,231	0,277	0,323	0,370	0,392785	2,705	3,246	3,787	4,328	5,410	6,492	8,656	12,983	21,639
Kraft_x	7,213	5,410	3,607	2,705	2,165	1,804	1,547	1,355	1,276	0,185	0,154	0,132	0,116	0,092	0,077	0,058	0,038	0,023
Kraft_y	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
Kraft	7,213	5,410	3,607	2,705	2,165	1,804	1,547	1,355	1,276	0,185	0,154	0,132	0,116	0,092	0,077	0,058	0,038	0,023

Remark: no cylinder is simulated with permeability of one.

Example 2: Force on conductor — as of page 50

File: Kraft_Leiter_x.xlsx

Körper 1		Abstand		**Körper 2**	
Radius_1	0,8509	2,626	0,8509	Radius_2	
Per_mü1	999		999	Per_mü2	
Leiter 1				**Leiter 2**	
Strom_1	1		-1	Strom_2	
L-ort x1	variabel		- - -	L-ort x2	
L-ort y1	0		0	L-ort y2	spiegelbildlich zu Leiter 1

	<<< zwischen den Zylindern >>>									<<< Leiter in positiver x-Halbebene >>>								
	0,15	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,85	1,25	1,5	1,75	2	2,5	3	4	6	10
x	0,069	0,092	0,139	0,185	0,231	0,277	0,323	0,370	0,392785	2,705	3,246	3,787	4,328	5,410	6,492	8,656	12,983	21,639
Kraft_x	7,401	5,668	4,026	3,334	3,088	3,184	3,720	5,192	6,831	-0,340	-0,022	0,047	0,066	0,071	0,066	0,053	0,037	0,022
Kraft_y	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
Kraft	7,401	5,668	4,026	3,334	3,088	3,184	3,720	5,192	6,831	0,340	0,022	0,047	0,066	0,071	0,066	0,053	0,037	0,022

Remark: normal case with two currents forward and back conductor

Example 3: Force on conductor with two forward currents — as of page 51

File: Kraft_Leiter_x.xlsx

Körper 1		Abstand		**Körper 2**	
Radius_1	0,8509	2,626	0,8509	Radius_2	
Per_mü1	999		999	Per_mü2	
Leiter 1				**Leiter 2**	
Strom_1	1		1	Strom_2	
L-ort x1	variabel		- - -	L-ort x2	
L-ort y1	0		0	L-ort y2	spiegelbildlich zu Leiter 1

	<<< zwischen den Zylindern >>>									<<< Leiter in positiver x-Halbebene >>>								
	0,15	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,85	1,25	1,5	1,75	2	2,5	3	4	6	10
x	0,069	0,092	0,139	0,185	0,231	0,277	0,323	0,370	0,392785	2,705	3,246	3,787	4,328	5,410	6,492	8,656	12,983	21,639
Kraft_x	-6,841	-4,905	-2,808	-1,552	-0,554	0,450	1,724	3,853	5,812	-0,523	-0,233	-0,161	-0,128	-0,096	-0,078	-0,058	-0,039	-0,023
Kraft_y	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
Kraft	6,841	4,905	2,808	1,552	0,554	0,450	1,724	3,853	5,812	0,523	0,233	0,161	0,128	0,096	0,078	0,058	0,039	0,023

Remark: two conductors on both sides of cylinder, back current via “earth”

Example 4: Force on conductor on y-axis

as of page 56

File: Kraft_Leiter_y.xlsx

Körper 1		Abstand		Körper 2	
Radius_1	0,8509	2,626	0,8509	Radius_2	
Per_mü1	999		999	Per_mü2	
Leiter 1				**Leiter 2**	
Strom_1	1		-1	Strom_2	
L-ort x1	0,2		-0,2	L-ort x2	
L-ort y1	variabel		- - -	L-ort y2	spiegelbildlich zu Leiter 1

	<<< im Zylinderbereich >>>									<<< Leiter in positiver y-Halbebene >>>								
	0	0,03	0,1	0,15	0,2	0,3	0,4	0,6	0,9	1,2	1,5	2	2,5	3	3,5	4	6	10
y	0,000	0,026	0,085	0,128	0,170	0,255	0,340	0,511	0,766	1,021	1,276	1,702	2,127	2,553	2,978	3,404	5,105	8,509
Kraft_x	3,213	3,164	2,742	2,319	1,909	1,265	0,849	0,410	0,156	0,062	0,021	-0,006	-0,014	-0,016	-0,016	-0,015	-0,018	-0,019
Kraft_y	0,000	0,399	1,170	1,512	1,693	1,752	1,616	1,207	0,706	0,420	0,270	0,163	0,126	0,111	0,103	0,097	0,083	0,098
Kraft	3,213	3,189	2,981	2,768	2,552	2,161	1,826	1,275	0,723	0,424	0,271	0,164	0,127	0,112	0,104	0,098	0,085	0,100
														Konvergenz ????				
< 50	0,000	0,000										0,004	0,000		0,076	0,033		0,077
< 100	0,000	0,000										0,000	0,000		0,012	0,006		0,040
< 150	0,000	0,000										0,000	0,000		0,001	0,002		0,079
< 200	0,000	0,000										0,000	0,000		0,000	0,000		0,026
	Kx	Ky										Kx	Ky		Kx	Ky		Kx

Remark: conductors are near the y-axis, please note the problem with convergence at high y-values, small v-values

Example 5: Force on cylinder

as of page 68

File: Kraft_Körper_.xlsx

Körper 1		Abstand		Körper 2	
Radius_1	0,8509	2,626	0,8509	Radius_2	
Per_mü1	999		999	Per_mü2	
Leiter 1				**Leiter 2**	
Strom_1	1		-1	Strom_2	
L-ort x1	variabel		- - -	L-ort x2	
L-ort y1	0		0	L-ort y2	spiegelbildlich zu Leiter 1

	<<< zwischen den Zylindern >>>									<<< Leiter in positiver x-Halbebene >>>								
	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,85	1,15	1,3	1,4	1,6	2	2,5	3	3,5	4
x	0,046	0,092	0,139	0,185	0,231	0,277	0,323	0,370	0,393	2,488	2,813	3,029	3,462	4,328	5,410	6,492	7,574	8,656
Kraft_x	-0,015	-0,063	-0,149	-0,285	-0,495	-0,827	-1,393	-2,525	-3,648	4,317	2,074	1,507	0,940	0,495	0,285	0,187	0,134	0,100

Remark: normal configuration with +/- current on both sides of cylinder.